ESTIMATING STACK GAS EMISSION RATES

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ABSTRACT

Municipal waste combustor emission rate estimates are needed for environmental impact and health risk assessments, setting operating permit limits, compliance certificates and other purposes. Preparing valid trace emissions estimates is not straightforward because the data frequently is not normally distributed and contains below detection limit results. From an emissions limit perspective, the concern is not the average emission rate, but rather a low limit with enough margin that test and CEMS averages (or individual run values) are unlikely to exceed the limit. Impact assessments must correctly characterize the emissions so that the conclusions are valid. This paper presents statistically valid methods for identifying underlying distributions, and estimating average emissions and associated limits given a specified exceedance frequency and statistical confidence level.

NOMENCLATURE

alpha = significance level; this is the statistical likelihood that an interval does not contain the parameter of interest

g_{CI} = coefficient for determining the Confidence Interval that contains the mean of the data set

g_{PIm} = coefficient for determining the Prediction Interval that contains the average of the next R replicate test.

g_{PIk} = coefficient for determining the Prediction Interval that contains the next k runs

g_{PIRT} = coefficient for determining the Prediction Interval that contains all the averages of the next T tests with R replicates each

g_T = coefficient for determining the Tolerance Interval that contains a specified percentage of all future replicates at a given confidence level

i = rank (first through N^{th}) of the point in a rank ordered data set

k = number of future tests to be included in an interval at a stated confidence level

n = number of runs conducted ($n = T*R$)

$t_{(n-1,\ 1-\mathrm{alpha})}$ = student t-distribution statistic for $n-1$ degrees of freedom and $1-$ alpha probability

CV = coefficient of variance calculated as the standard deviation divided by the mean

N = number of data points in the set

R = number of replicates in a test

P = proportion of future tests to be included in an interval at a stated confidence level

S_x = standard deviation of x

Sg_x = geometric standard deviation of x; computed by exponentiation of the standard deviation of the natural logarithm of x

T = number of tests conducted

UL = either the single detected value or detection limit used in Eq. (4)
$X^2_{(n-1, \text{alpha})}$ = chi square statistic for $n - 1$ degrees of freedom and alpha probability
Z_p = distance in standard deviations that the boundary on the normal distribution is located from the average to encompass probability p
$\langle x \rangle$ = mean (arithmetic average) of x
$\langle x \rangle_g$ = geometric mean of x; computed by exponentiation of the arithmetic average of the natural logarithm of x
$\% f_i$ = median rank or quantile of the i^{th} data point

INTRODUCTION

Many engineers are familiar with some statistical concepts. They frequently do not know, however, that the commonly taught statistical procedures assume that the data is taken randomly from a normally distributed population. When these assumptions are satisfied, the mean (arithmetic average) and standard deviation completely describe the data, and intervals can then be determined which are likely to contain: the data average (Confidence Limits); the results of a specified number of future tests (Prediction Limits); or a specified proportion of all future test results (Tolerance Limits). These limits provide answers (with a known degree of certainty) to the following questions:

(a) What limit is likely to bound the next test average?

(b) What limit is likely to bound the largest individual run in a future series of tests?

(c) What limit is likely to bound the averages for a specified number of future tests?

(d) What limit is likely to contain a specified percentage of all future runs or test averages?

In order for statistical intervals to correctly answer these questions, the data must either be, or be made to be, normally distributed. The averaging time or number of replicates must also be properly accounted.

The examples in this paper use trace metals emissions data (Appendix A) from testing conducted at the Southeast Resource Recovery Facility (SERRF) in 1988 and 1989 (TRC) and 1990 (Bell, et al.). The SERRF facility in Long Beach, California has three nominal 460 tons per day (TPD) L&C Steinmuller mass burning waterwall boilers equipped with Selective Non-Catalytic Reduction (SNCR) and flue gas recirculation (FGR) NO_x control, a slaked lime rotary atomizer based spray dryer absorber and reverse air baghouse for acid gas and particulate control.

HAVING CONFIDENCE A LIMIT WILL BE MET

Upper statistical limits provide a level below which the parameter of interest is likely to be found with a known statistical confidence or probability. That is, if a test is repeated many times and the upper limit is calculated at the 5% significance (95% confidence) level for each test series, then the true value will be below the calculated upper limit 95 out of 100 times. It is important to remember that the performance of the plant does not change as long as the plant is operating normally. What changes with each test or data analysis is the estimate of the interval because each estimate is based on available data. There is always a possibility that a calculated limit will be exceeded when nothing has changed from the base data period simply due to unfortunate coincidence.

Statistical intervals define the exceedance risk based on measured performance. Different types of intervals define the limits for different averaging times and monitoring requirements.

In general, upper limits are determined by adding the product of an interval coefficient ($g_{-\,-}$) and the standard deviation to the mean as shown in Eq. (1).

$$\text{interval} = \langle x \rangle + g_{-\,-} S \tag{1}$$

If the data has been transformed to achieve normalcy before calculating the mean and standard deviation, the limit calculation should use the transformed mean and standard deviation. The Eq. (1) result is then mathematically inversed to express the limit in original data units.

Upper limits are provided, rather than estimates of bands in which the data is likely to reside, because these are limits above which measured emissions are not likely to be found. If the lower limit is desired, simply subtract the product from the mean.

When deciding how much confidence to have in a limit or margin to provide, it is important to balance the cost of an error and the likelihood of setting an excessively high limit. If an individual test exceeds a limit, there can be two causes: first, the result may simply be a statistical aberration; or second, there may be something wrong with the facility being tested. If an examination of the plant indicates that there is nothing wrong, then retesting should yield a result that is below the limit. Thus, the cost of a tight limit may simply be the price for retesting. On the other hand, if there is a perception that a limit should never be exceeded, then higher confidence levels and periods between likely exceedances are indicated. There are costs to increasingly higher limits. From a project perspective, higher limits create a perception of much larger emissions than actually will occur. From a regulatory perspective, higher limits consume greater amounts of the pollutant increment in attainment areas and require more offsets in nonattainment areas.

As a general rule, 95% confidence limits balance the types of statistical errors encountered and 99% confidence limits leave only a 1% chance of finding noncompliance when the plant is really meeting a limit. g_{--} factors for five different types of intervals are provided in Appendix C-1 and C-2 for the 95% and 99% statistical confidence levels. The formulas in Appendix B can be used to calculate the factors for other confidence levels and conditions.

Confidence Interval for the Mean

The upper Confidence Limit for the mean defines a level that is likely to be above the population mean. It is not an estimate of how large an individual test run or average might be found during any specific test. Appendix C contains the coefficients (g_{CI}) needed to estimate the upper Confidence Limit for the mean.

If the problem is to estimate a limit below which long-term average performance (the population mean) is likely to be found, a Confidence Limit is used. If the question being addressed refers to average values, such as multi-hour block-average CEMS results, then the block average standard deviation should be used in Eq. (1). This standard deviation can be estimated using the factors provided in Appendix C-3. Appendix C-4 provides standard deviation adjustment factors for different averaging times where the samples are randomly drawn and not block-averaged. This limit applies to estimating average annual or lifetime facility emissions based on facility data; it also applies to average emissions from a group of facilities that have been tested and shown to be similar provided the test data is pooled prior to estimating the Confidence Interval.

Correcting test data for sampling (averaging) time is important if underestimated limits are to be avoided. For example, if 8-hr sampling time data is used to establish a minimally acceptable 4-hr permit limit, as discussed in detail later, the standard deviation used in Eq. (1) will be underestimated by 25–30%.

Tolerance Limits that Contain a Specified Data Percentage

If an interval is needed in which a specified percentage of the test results are likely to reside, a Tolerance Limit is calculated (use g_T).

The Tolerance Limit is useful for estimating the emissions of a new source. Historical data provide a basis for saying "this is how plants like the proposed facility perform." The Tolerance Limit says that, based on the data, we are statistically confident (say 95%) that a given percentage (say 99%) of all future plants of this type will exhibit emissions less than the Tolerance Limit. Thus, we have a given level of confidence that the emissions from a proposed plant will be less than the Tolerance Limit calculated from the performance of several similar, existing facilities.

Prediction Limit for the Next Tests

If a limit estimate is needed to encompass the average result of the next set of test runs, the upper bound Prediction Interval for the Test Mean is the correct limit (use g_{PIm}). This upper bound applies when you are only interested in the results of the next test series being below a limit (e.g., a contractor with a single acceptance test responsibility).

The Prediction Interval for k Future Runs (use g_{PIk}) provides an upper bound above which any of the individual results for k future runs are unlikely to be found. This interval estimate provides a specified level of certainty that all scheduled runs are likely to be below the limit. This applies to a situation where a regulatory agency sends out test crews to perform a single run every year and no exceedances are desired over some future period of time.

The Prediction Interval for the Individual Averages of T future, R Replicate tests (g_{PIRT}) provides an upper bound estimate for the averages of a specified number of future tests which have the same number of replicates. This interval applies when a facility permit requires periodic testing and no exceedances are desired between permit renewals or over some longer period such as bond life or operating contract term.

CORRECTLY DESCRIBING THE DATA

The average and standard deviation must describe the data for the calculation to provide the expected certainty that the limits will not be exceeded. This means that the data analysis must discern the nature of the underlying distribution, transform the data as necessary to make it normal, and properly handle Below Detection Limit (BDL) test results.

Structuring the Data for Analysis

Data analysis begins with data tabulation. The data must be correctly transcribed from its original source and be in consistent, diluent corrected units for the statistical results to be meaningful. To help understand the data, it is a good idea to include plant operating conditions (number of baghouse modules or electrostatic precipitator fields in use, scrubber reagent flows, etc.), key temperatures and pressures and any unusual events such as hopper pluggages or intentional off-design operation. That way, physical and chemical explanations may be able to be developed for any apparently unusual results and nonrepresentative results may be identified and excluded from the analysis.

Data analysis is facilitated if the detection limit is preceded by a minus sign when BDL results are recorded. This data coding method should not create a presentation problem since most spreadsheets can be set to display negative numbers in parentheses. Unlike the conventional "<" sign, computers readily ignore the ( ) display and isolate negative numbers. This BDL coding method is particularly useful if arithmetic averages using half the detection limit for all BDL are desired. A simple mathematical test can be written into a spreadsheet to create a transformed data set made up of the original value for all positive results and half the absolute value of negative results. The arithmetic average of this transformed data set fulfills a half detection limit for BDL results.

Underlying Distribution

Before accepting that the arithmetic average and standard deviation correctly characterize the data, it is necessary to examine the data for normalcy. This is particularly important if limit calculations which use the mean and standard deviation are to be physically meaningful as well as mathematically correct. If normal statistics are applied to a log normal data set, for example, the result can be an unachievable permit limit and incorrect assessment of the environmental impacts and health risks associated with the emission.

The EPA recommends in SW-846 calculating the data mean and variance (standard deviation squared) and comparing the results. If the variance is larger than the mean, the data is probably not normally distributed and transformation prior to comparison to standards is recommended.

This test assumes that the average and standard deviation can be directly calculated because there are no BDL results and identifies severely nonnormal data sets. A more powerful method that automatically accommodates BDL results is the use of probability plots. Probability plots are prepared by rank ordering the data, determining the percentage median rank or quantile associated with the rank of each data point and plotting the data against percentage quantile on special probability paper. If median rank tables (Lipson and Sheth, 1973) are not available, the plotting position ($\%f_i$) can be estimated using Eq. (2).

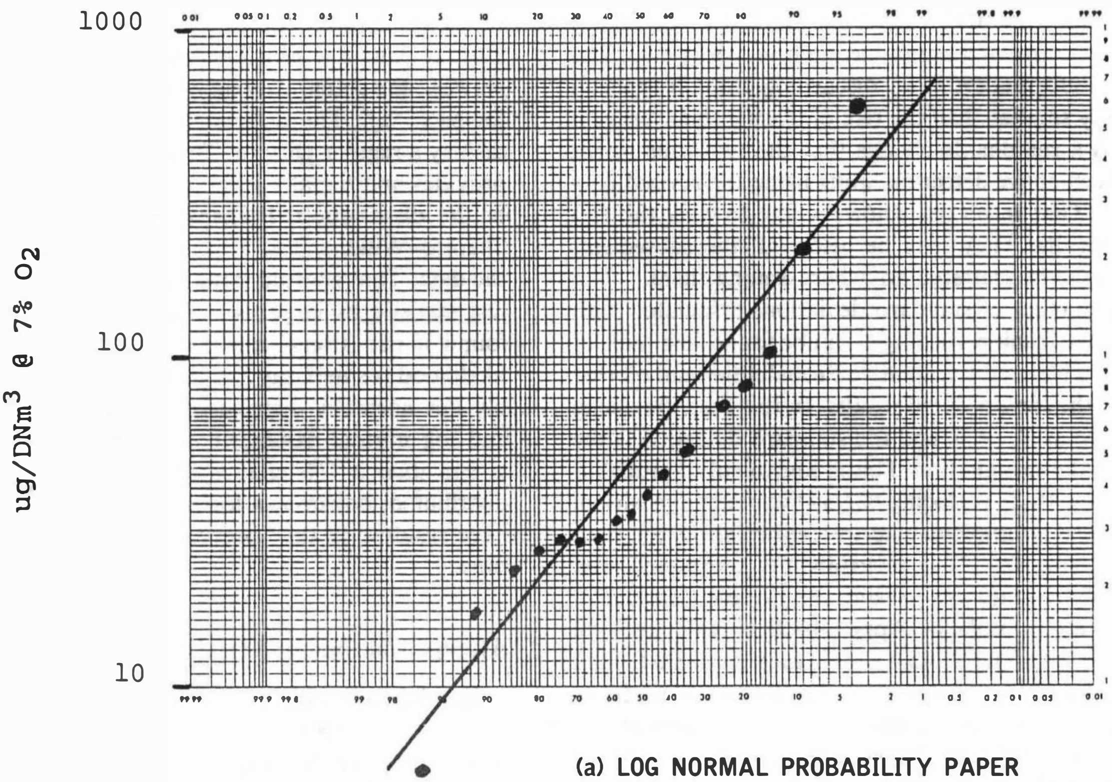
$$\%f_i = 100 * (i - 0.375)/(N + 0.25) \qquad (2)$$

The total number of runs (both those with detected and BDL results) are used to establish N.

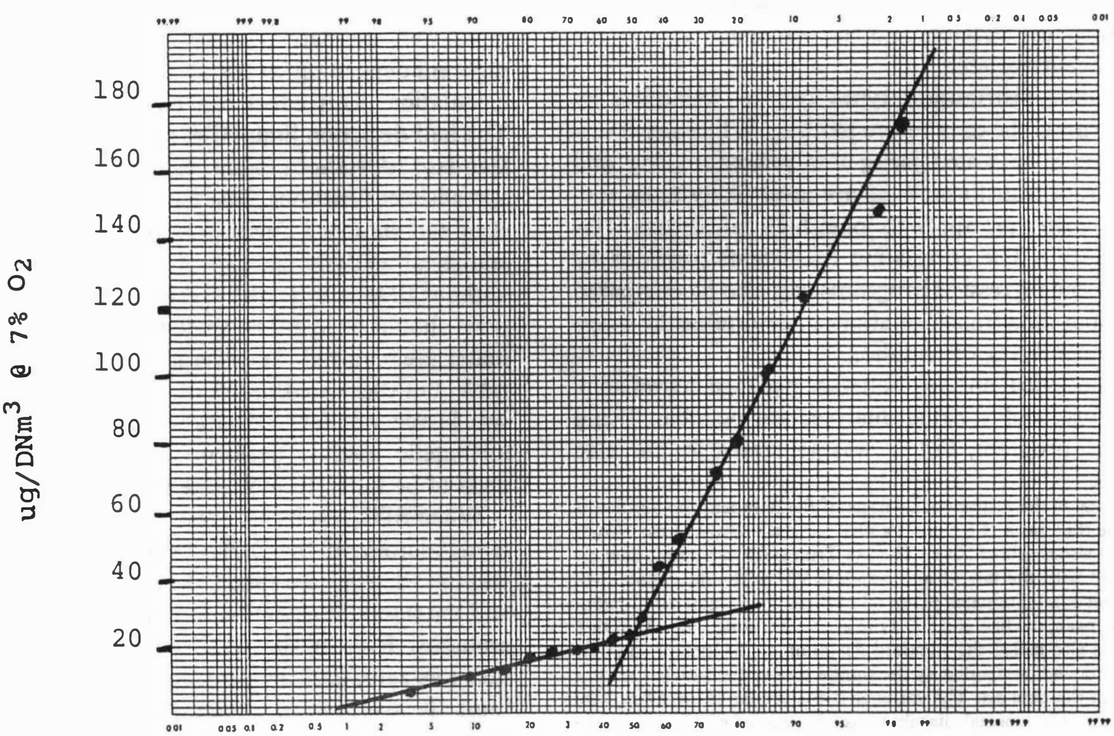
Normally distributed data plot as a straight line on normal probability paper. The average is located at the 50% probability point and the standard deviation is one third of the difference between the values plotting at 7% and 93%. If the line is not straight, other types of distribution paper can be tried. Figure 1(a) is a log normal probability plot and Fig. 1(b) is a normal probability plot of the zinc emissions data from SERRF. Note that the plot shows a distinct kink on the normal probability paper and a relatively straight line on the log normal probability paper. This provides evidence that the data is not normally distributed but may be log normally distributed.

BDL data indicates that part of the distribution is censored. The number of BDL results locate the plotting position of the detected results; but they do not contribute any direct information about the expected value of specific individual censored points. The detection limit is usually plotted for BDL data points since the expected value of the rank-ordered BDL results could be distant from the detection limit.

Detection limits occasionally change over time as sampling and laboratory methods improve. Consequently, a plant emissions data set made up of several test reports may contain multiple censoring (detection) levels. For multiple censored data sets, the cleanest thing to do is to treat all results smaller than the highest detection limit as missing. If run duration was changed, however, to improve the detection limit, then BDL results from the original testing above the new detection limit contribute no information and should be discarded. For example, consider a data set where the first series of three test runs results in only one above detection limits result, so the test duration was extended for all subsequent runs. If the longer sampling time results in essentially all above detection limits



(a) LOG NORMAL PROBABILITY PAPER



(b) NORMAL PROBABILITY PAPER

FIG. 1 PROBABILITY PLOTS OF THE ZINC EMISSIONS DATA results, the two BDL runs from the first series should probably be treated as not having been performed. The loss of information from discarding the two runs is probably smaller than the risk of biasing the average if there is a reasonable chance that the two runs might have been above detection limits using the revised procedure. That is, no one knows where to rank the two BDL results. They may be below all the subsequent test results or scattered anywhere among them.

The number of standard deviations associated with the median rank percentages can be found in a table of Standard Normal Cumulative Probabilities and plotted against the rank ordered results. In this case, the mean is located at $Z_p = 0$, and the standard deviation is the slope of the straight line. This is equivalent to probability plots with Z_p replacing $\%f_i$ as the plotting position. It can also be implemented in a spreadsheet program by first using Eq. (2) to determine the percentage rank order of the data points and the inverse cumulative probability formula in Appendix B to calculate the associated Z_p. The normal plot is then generated using the X–Y graphing feature found in many spreadsheets by plotting each data point against its associated Z_p. From the spreadsheet's linear regression procedure, the mean is the intercept and the standard deviation is the slope of the line calculated using the data points as the dependent variable and the associated Z_p's as the independent variable.

Different distribution types can be identified by transforming the data before plotting. While any transform could be tested using this procedure, log and square root transformations are most likely to be successful since they apply to long, high tailed data (Natrella, 1966). The correlation coefficients provided by most linear regression programs can also be compared for various data transformations to determine the shape of the underlying distribution.

If a statistical analysis software package is available, then more sophisticated analyses are possible. Using one such program, Fig. 2[1] was generated for the zinc emissions data. In addition to normal plots, detrended normal plots are provided which show the deviation between the actual data point and the theoretical value for each $\%f_i$. The detrended normal plot should not display a regular pattern. This examination of the data provides additional information on the characteristics of the distribution. Perhaps most important are the Shapiro–Wilks and K–S (Lilliefors) normalcy tests to the data. The reported significance is the chance that a test statistic as large as calculated would have been found if the (transformed) data were normal. If the likelihood is small, say less than 5%, the data is probably not normally distributed. Comparing the normal and log normal assumptions for the zinc data, there is less than a 1% chance that the data is normally distributed; however, a log normal distribution would have resulted in the observed data set about half the time.

Average and Standard Deviation

Hand calculators, spreadsheet programs and statistical analysis packages usually calculate averages and standard deviations. Typically, only statistical analysis packages use special mathematical algorithms to avoid machine rounding errors. As a result, an estimated mean and standard deviation is likely to be slightly different depending on the equipment used to make the calculation. This effect can be seen in Appendix A. The half detection limit BDL substituted averages and standard deviations were calculated using internal spreadsheet functions. The Normal Distribution averages and standard deviations were calculated using a statistical analysis package. For the trace metals sets without BDL results, the averages and standard deviations should be identical since no substitutions or estimates are involved. The averages are generally comparable, but there are third digit differences (for this data set) in the standard deviations. Even with the 64 bit arithmetic used to perform these calculations, steps need to be taken to minimize rounding errors.

When BDL results are present, correct data handling is mandatory. If the BDL results are simply ignored and the "detects" averaged, the average will be too high and the standard deviation too small, both by unknown amounts. Even if the BDL results are included but taken as zero, half the detection limit or the detection limit, the average and standard deviation remain biased by an unknown amount. The best approach is to esti-

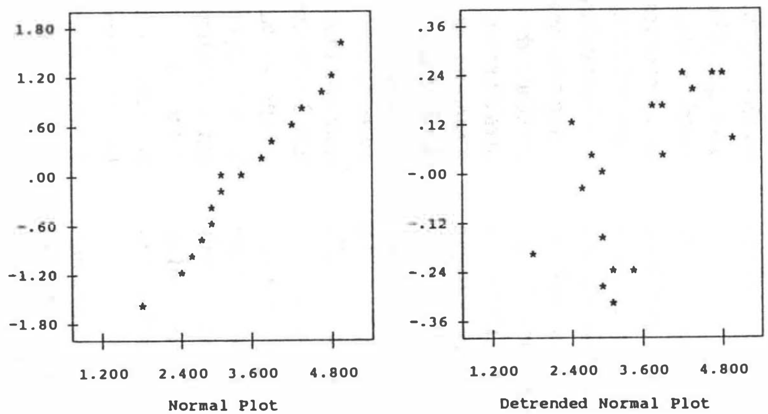
[1] The Mean is the arithmetic average of the data and the Median is the value that has a $\%f_i$ equal to 50% (or the average of the closest values on either side of 50%). The 5% Trim is the arithmetic average computed with the largest and smallest 5% of the data discarded. This procedure eliminates potential outliers. For a normal distribution, the mean, median and 5% Trim are all the same. The STanDard ERRor is also called the standard deviation of the mean and is the standard deviation divided by the square root of the number of runs (Cases) in the data set. The variance is the square of the STanDard DEViation. For normal distributions, the variance is less than the mean. The Min., Max., and Range are the smallest and largest data values and the difference between the largest and smallest value. The InterQuartile Range indicates where the center half of the data resides. Skewness is the third moment of the data distribution and generally has a value less than 0.5–1.2 (depending on statistical significance and number of runs) for a normal distribution. S E Skew provides an estimate of the standard error for the skewness. The Kurtosis is the fourth moment of the data and S E Kurt is the standard error of the kurtosis.

SPSS/PC+ 7/23/91

ZN

Valid cases: 18.0 Missing cases: 30.0 Percent missing: 62.5

Mean	3.4859	Std Err	.2096	Min	1.8668	Skewness	.1453
Median	3.2305	Variance	.7906	Max	4.9983	S E Skew	.5363
5% Trim	3.4918	Std Dev	.8892	Range	3.1315	Kurtosis	-.9029
				IQR	1.4109	S E Kurt	1.0378



	Statistic	df	Significance
Shapiro-Wilks	.9596	18	.5656
K-S (Lilliefors)	.1542	18	> .2000

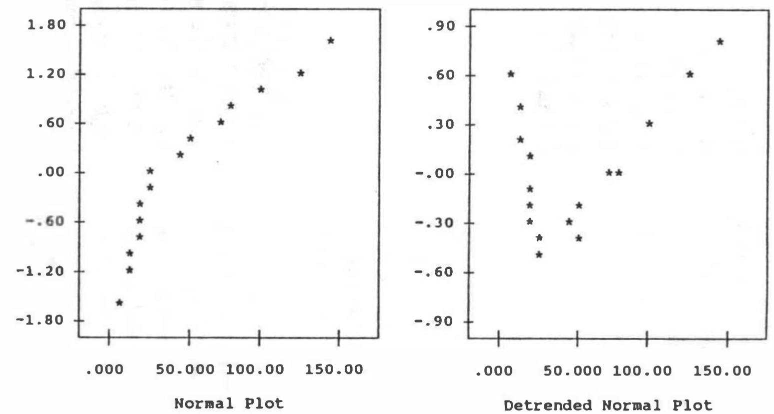
(a) NORMAL DISTRIBUTION ASSUMED

SPSS/PC+ 7/23/91

ZN

Valid cases: 18.0 Missing cases: 30.0 Percent missing: 62.5

Mean	46.9582	Std Err	9.8088	Min	6.4674	Skewness	1.2707
Median	25.4116	Variance	1731.816	Max	148.1614	S E Skew	.5363
5% Trim	43.5853	Std Dev	41.6151	Range	141.6940	Kurtosis	.7439
				IQR	55.8338	S E Kurt	1.0378



	Statistic	df	Significance
Shapiro-Wilks	.8325	18	< .0100
K-S (Lilliefors)	.2323	18	.0112

(b) LOG NORMAL DISTRIBUTION ASSUMED

FIG. 2 STATISTICAL ANALYSIS COMPUTER PROGRAM EXAMINATION OF THE ZINC EMISSIONS DATA mate the likely values for BDL data using the procedure described in Rigo (1989), Travis and Land (1990), and Helsel (1990). Expected values for the rank ordered BDL results are estimated using the mean, standard deviation determined during the distribution checking procedure and Z_p associated with each BDL value using Eq. (3):

$$BDL_{est} = \text{mean} + Z_p * \text{standard deviation} \quad (3)$$

If the data was transformed before determining the mean and standard deviation, inverse the transformation after calculating BDL_{est}. For example, square BDL_{est} if the data was square root transformed before analysis; calculate the exponential of BDL_{est} if the natural logarithm of the data was analyzed.

The validity of the transformation and filling process can be determined by repeating the normal plotting and distribution analysis procedures with the filled data set. If the correct underlying distribution has been used to calculate BDL_{est}, then the estimated data should appear to be part of the complete data set. If an incorrect underlying distribution has been assumed, then a distinct kink or bowing in the plot will demark the estimated to measured data transition. If, however, policy requires the use of an arithmetic average regardless of the nature of the underlying distribution, then the filling process will result in the best estimate that can be made for the data average (Helsel, 1990).

If the data is made up of two distinct groups and the data is not partitioned before analysis, the calculated standard deviation may be seriously in error. Limit or emission rate projections based on probabilities outside the range of $\%f_i$ covered by the data are likely to be as much as three orders of magnitude high for data sets made up of near detection limits data and high values. Data grouping is necessary if the normal plots indicate that two straight lines are present, especially if they appear to be off-set rather than intersecting. If the data appears to be best described by two straight lines, the low value group should be regressed against Z_p while treating the data in both the high value group and BDL as missing to obtain estimates for the mean and standard deviation of the low value data group. Some of the trace emissions data in Appendix A exhibits this characteristic. Means and standard deviations used to estimate the Upper Tolerance Limit provided in Appendix A were based on the group data.

While the intuitive validity of the analysis declines as more than half the data is censored, the mean can be calculated if an estimate of the coefficient of variance can be obtained from previous tests or by assuming that a particular material behaves like another for which coefficients of variance are available. For example, if the data set contains information on a relatively volatile metal like lead, that coefficient of variance could be used to estimate the mean of another relatively volatile metal like arsenic where only one run might have been above detection limits. The estimate is made using Eqs. (4) and (5):

$$\langle x \rangle = UL/(1 + Z_{pl} * CV) \quad (4)$$

$$S = CV * \langle x \rangle \quad (5)$$

If there are no detected values, applying the above equation to detection limits yields an upper bound estimate for the mean. If the mean is actually any larger than the Eq. (4) value, there should have been at least one above detection limit result. Of course, the mean might be substantially less than the estimate or the material completely absent.

Averaging Time and Number of Replicates

The average emission rate for a process is not affected by the length of sampling or the number of replicates averaged. This is because the average (arithmetic, geometric mean, etc. as appropriate) is the same whether we average results that are themselves made up of averages of data subsets or all the data is individually used.

The standard deviation, however, changes with averaging time or number of samples that are averaged. To illustrate the point, a simulation can be done which uses a known mean and standard deviation to generate a random set of data points with that mean and standard deviation. Taking the individual data points as "1 hr" data, the effect of 3-, 4-, 8- and 24-hr sampling or averaging times can be shown by making up additional data sets that average a number of hours together. Figure 3 is a normal probability chart displaying those results. As can be seen, the mean remains unchanged, but the slope (standard deviation) changes with different averaging times.

One of the benefits of having a normal distribution, according to statistical theory (Mason, et al., 1989), is that averages of random samples are also normally distributed with the same mean as the parent population, but the standard deviation is reduced by the square root of the number of samples used in each average. This means that if we know the mean and

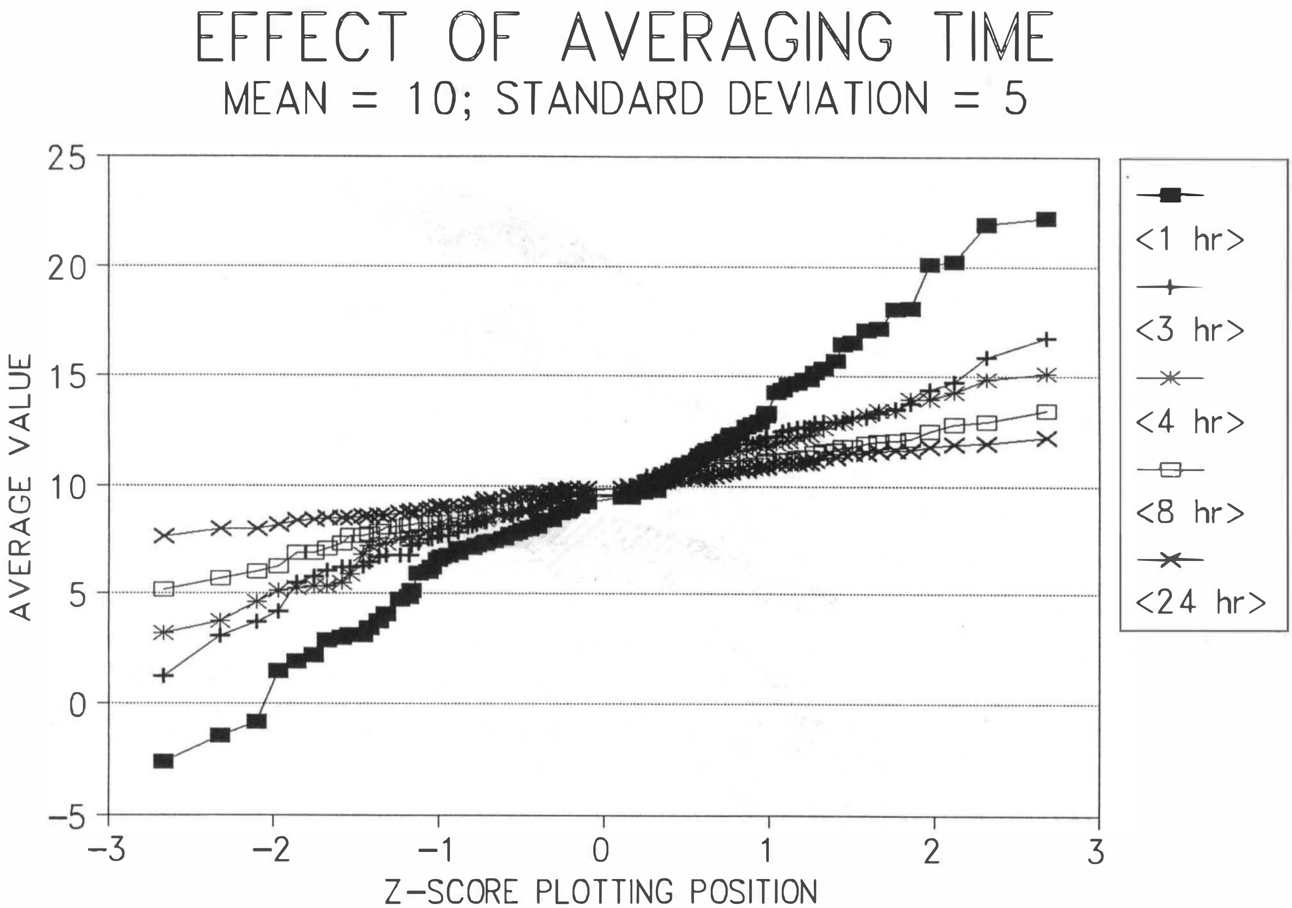


FIG. 3 NORMAL PLOT OF THE EFFECT OF DIFFERENT AVERAGING TIMES ON THE DISTRIBUTION OF RESULTS

standard deviation for a given sampling time (say 8-hr samples) and we want to compare the results to a 3-hr standard, the average can be directly compared. But, if the intention is to develop a not to be exceeded limit or standard, then the standard deviation must be corrected for sampling time or number of replicates before calculating the limit.

While statistical theory says that averaging time standard deviations should be related by the square root of the number of samples involved or sampling period, an exponent of 0.4 instead of 0.5 provides a more consistent estimate of the change in standard deviation when hourly data is block averaged. For example, if sixteen 8-hr duration test results are available, the mean is the same as for a 3-hr test duration, but the standard deviation for the 3-hr tests is 1.5 times as large as that apparent from the test data. If the averaging time were to be reduced to a one hour basis that matches dispersion modeling emission rate input requirements, then the standard deviation increases to 2.3 times that which is apparent from the 8-hr data.

In addition, while the standard deviation of the underlying distribution is constant, the data based estimate is exactly that, an estimate. To be conservative, it is advisable to determine the Upper Confidence Limit for the standard deviation and use that value in lieu of the data estimate when correcting for differences in sampling times or number of replicates. For the previous example, the 1-hr standard deviation is 3.3 times the 8-hr average standard deviation, including the uncertainty of the standard deviation as well as the averaging time effect.

Tabulations of the factors for making this correction are provided in Appendices C-3 and C-4 for the 0.4 power and square root of the number of samples respectively. Two sets of factors are provided to enable the conversion of data with a sampling time or number of replicates greater than 1, to a 1-hr (single run) base

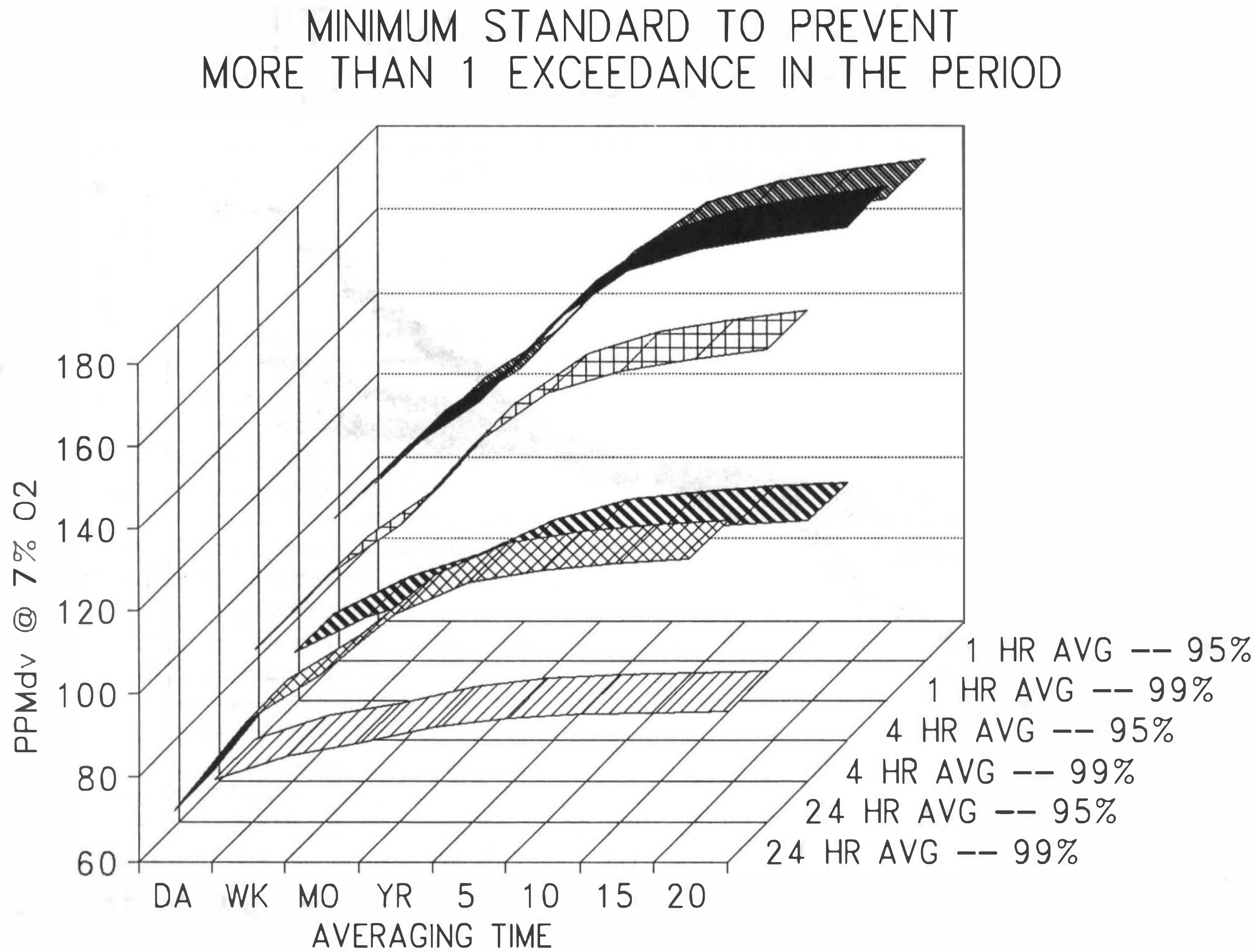


FIG. 4 UPPER TOLERANCE LIMITS CORRESPONDING TO ONE EXCEEDANCE PER CALENDAR PERIOD AS A FUNCTION OF BLOCK AVERAGING TIME AND STATISTICAL CONFIDENCE LEVEL

and to go from a 1-hr averaging time basis to longer averaging times. These factors should be used in converting stack sampling results into hourly emission rate data for use in ambient dispersion analysis if upper limits are used. They can also be used to estimate compliance limits for 4-, 8- and 24-hr Continuous Emissions Monitoring System averaging times given 1-hr data.

The effect of averaging time can be seen in Fig. 4. Figure 4 shows the upper Tolerance Limit that corresponds to $\%f_i$ for a given frequency of exceedances based on the 1-hr log normal mean (4.243 log units) and standard deviation (0.192 log units) for Carbon Monoxide (PPMdv @ 7% O_2) emissions at SERRF. $\%f_i$ was determined using Eq. (1) for i equals 1 and the denominator equal to the number of averaging time periods in the calendar period. The calculated limits are provided in Appendix D for 95% and 99% confidence levels based on both normal and log normal distribution assumptions. The standard deviation was also corrected for block averaging times.

It is important to realize that in this analysis the process creating the emissions is assumed to be stationary. That is, the fundamental process causing the emissions does not change and its long term performance is characterized by the mean and standard deviation. The achievable emission limit changes, however, depending upon how often an exceedance is acceptable.

Given that the average Carbon Monoxide emissions are about 71 PPMdv @ 7% O_2, if only one exceedance every 20 years is desired at the 95% confidence level, the permit limit needs to be 172 PPMdv @ 7% O_2 (about $2\frac{1}{2}$ times the average) to avoid violations due to the inherent variability in the process. If the averaging time is increased to 24 hr, the permit limit only needs to be 86 PPMdv @ 7% O_2 (about 1.2 times than the average).

Both these limits apply to the same process! The only difference is the averaging time. Further, since there is always some chance that a properly operating process will naturally generate a high emission rate, it is prudent that limitations either spell out an acceptable exceedance frequency (like the "second high" limitation found in some national primary and secondary ambient air quality standards) or the limit include a margin for statistical aberrations.

The importance of using the proper underlying distribution is emphasized by the Carbon Monoxide CEMS limit example. If the CEMS data were treated as being normally distributed instead of log normally distributed, the 24-hr averaging time limit is practically unaffected (due to operation of the central limit theorem), but the 1-hr averaging time limit would have been 30 PPMdv @ 7% O_2 understated. Improper data analysis can lead to the specification of a limit that will be exceeded much more than expected. Reviewing the tables in Appendix D shows that choosing to be 99% instead of 95% confident in the limit has a small effect.

CONCLUSION

The methods provided enable the correct characterization of emissions test data. Data taken under one set of test conditions (run duration and number of runs in a test) can be adjusted to other test conditions. Limits which are unlikely to be exceeded at a given statistical confidence level can be determined.

These methods enable anyone setting limits to estimate the likelihood of exceeding a specified limit when there is nothing wrong with the plant. This can result in correctly wording permits to preclude the appearance of permit violations when noncompliance has not actually occurred.

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APPENDIX A

APPENDIX A-1 TRACE METALS EMISSIONS TEST RESULTS FOR SERRF

DATE	UNIT	LOCATION	STEAM FLOW kLb/hr	SAMPLE TEMP F	SAMPLE SIZE DSCF	SAMPLE DURATION MINUTES	SAMPLE MOISTURE %	SAMPLE OXYGEN %	SAMPLE CO2 %	Fo
09-Nov-88	1	STACK	120.3	280	128.1	180	18.0	10.4	8.6	1.22
09-Nov-88	1	STACK	119.3	285	125.7	180	17.7	9.8	8.6	1.29
09-Nov-88	1	STACK	121.4	290	118.9	180	13.6	11.0	8.4	1.18
07-Aug-90	1	STACK	114.5	311		480		9.4		
09-Aug-90	1	STACK	116.4	299		480		9.2		
14-Aug-90	1	STACK	116.1	285		480		9.3		
06-Dec-88	2	STACK	120.1	332	176.2	180	16.3	10.2	8.9	1.20
06-Dec-88	2	STACK	121.4	331	172.7	180	14.3	9.9	9.2	1.20
05-Apr-89	2	STACK	4.6	298	169.8	180	18.0	10.2	8.9	1.20
20-Aug-90	2	STACK	116.2	285		480		10.7		
20-Aug-90	2	STACK	116.2	285		480		10.7		
22-Aug-90	2	STACK	117.1	285		480		9.0		
03-Apr-89	3	STACK	107.8	320	373.0	480	16.3	10.8	8.2	1.23
04-Apr-89	3	STACK	123.1	325	355.5	480	16.9	9.6	8.6	1.31
05-Apr-89	3	STACK	120.1	312	360.1	480	16.9	10.0	9.2	1.18
24-Aug-90	3	STACK	116.8	283		480		10.6		
24-Aug-90	3	STACK	116.9	283		480		10.6		
30-Aug-90	3	STACK	117.1	283		480		9.4		

DATE	UNIT	Sb ug/DNm3 @ 7% O2	As ug/DNm3 @ 7% O2	Ba ug/DNm3 @ 7% O2	Be ug/DNm3 @ 7% O2	Cd ug/DNm3 @ 7% O2	Cu ug/DNm3 @ 7% O2	Pb ug/DNm3 @ 7% O2	Mn ug/DNm3 @ 7% O2	Ni ug/DNm3 @ 7% O2	Se ug/DNm3 @ 7% O2	Zn ug/DNm3 @ 7% O2	P ug/DNm3 @ 7% O2	Tot Cr ug/DNm3 @ 7% O2	Cr VI ug/DNm3 @ 7% O2	Hg ug/DNm3 @ 7% O2
09-Nov-88	1	(33.732)	(33.732)	2.108	(6.746)	12.228	3.373	1.602	(6.746)	3.373	(33.732)	80.114	13.071	5.482		168.7
09-Nov-88	1	(32.515)	(32.515)	(6.503)	(6.503)	10.974	1.219	(32.515)	(6.503)	2.032	(32.515)	11.787	101.609	1.219		203.6
09-Nov-88	1	(38.551)	(38.551)	11.083	(7.710)	39.033	2.409	(38.551)	(7.710)	4.337	(38.551)	50.598	5.783	2.409		253.0
07-Aug-90	1		(0.012)		(0.007)	0.969	2.041	1.026	0.531	2.234	(0.372)	22.945				
09-Aug-90	1		0.029		(0.007)	0.247	2.426	0.131	0.416	2.033	(0.364)	12.961				
14-Aug-90	1		(0.012)		(0.007)	0.116	1.187	0.180	11.897	0.552	(0.385)	16.670				
06-Dec-88	2	(32.730)	(32.730)	7.844	(7.574)	(8.926)	8.926	32.730	7.844	7.844	(32.730)	122.264	22.451	7.844		245.9
06-Dec-88	2	(34.867)	(34.867)	9.588	(8.136)	(9.588)	8.136	34.867	9.588	8.426	(34.867)	44.164	35.448	7.554		248.7
05-Apr-89	2	(36.762)	(36.762)	10.110	(8.578)	(8.884)	9.191	3.676	5.514	5.514	(27.572)	27.878	11.335	7.965		257.0
20-Aug-90	2		0.064		(0.008)	0.143	1.469	1.686	3.849	0.979	(0.042)	21.897				
20-Aug-90	2		(0.012)		(0.008)	0.174	2.326	1.550	0.884	0.517	(0.039)	18.633				
22-Aug-90	2		(0.012)		(0.007)	0.173	1.729	2.465	8.025	0.491	(0.012)	51.745				
03-Apr-89	3	3.719	(5.469)	0.984	(1.094)	2.516	2.734	16.078	32.812	0.219	(5.469)	71.750	33.687	0.328		137.0
04-Apr-89	3	1.492	(5.737)	0.344	(1.147)	0.574	2.754	10.901	367.641	0.115	(5.737)	100.172	190.361	0.115		143.3
05-Apr-89	3	5.664	(5.664)	1.699	(1.133)	2.265	4.757	30.471	284.995	0.113	(5.664)	148.161	201.626	0.227		116.6
24-Aug-90	3		(0.015)		(0.009)	0.135	2.707	0.875	3.031	1.104	(0.044)	18.587				
24-Aug-90	3		(0.013)		(0.008)	0.358	1.872	1.158	11.233	6.021	(0.043)	18.453				
30-Aug-90	3		(0.013)		(0.008)	0.046	1.870	0.290	12.006	8.929	(0.013)	6.467				
HALF D.L. FILL		------	------	------	------	------	------	------	------	------	------	------	------	------	------	------
AVERAGE		3.625	6.286	5.224	1.353	4.647	3.396	9.734	42.819	3.046	6.060	46.958	68.375	3.683	0.204	197.091
STD DEV		1.705	8.046	4.109	1.752	9.261	2.578	12.027	104.086	2.676	7.693	40.370	73.309	3.291	0.166	53.330
UTL-95%		19.937	26.724	17.508	5.803	28.170	9.944	40.284	307.195	9.843	25.601	149.497	287.567	12.043	0.700	356.547
NORMAL DIST.																
AVERAGE		3.625	-0.072	4.469	0.759	1.795	3.396	6.042	19.826	3.046	14.754	46.958	68.375	2.061	0.204	189.251
STD DEV		1.705	0.074	5.360	4.296	10.167	2.606	13.576	109.383	3.052	13.075	41.615	77.756	2.973	0.177	50.257
UTL-95%		19.937	0.117	20.496	11.671	27.620	10.015	40.524	297.659	10.799	47.965	152.661	300.864	9.611	0.732	339.518
LOGNORMAL DIST.																
AVERAGE		3.155	0.003	7.576	0.075	0.338	2.744	11.023	47.040	1.478	8.604	32.652	35.502	0.524	0.136	182.509
STD DEV		1.745	5.461	1.350	2.892	13.045	1.881	2.226	2.138	4.224	0.641	2.433	2.121	6.442	2.776	1.328
UTL-95%		651.639	0.216	15.510	1.113	230.312	13.654	77.015	298.211	57.448	2.782	312.457	335.998	59.484	2.879	426.138

NOTE: DETECTION LIMITS SHOWN IN ( ) FOR BDL RESULTS

APPENDIX B

The equation for the Prediction Interval for T future R Replicate tests is by this author following the suggestion in Hahn (1969). The balance of the equations for estimating statistical intervals were brought together by Hahn (1970a, 1970b). The equations for estimating Z_p, $t_{(n-1,\ alpha)}$, and $X^2_{(n-1,\ alpha)}$ were taken from Abramowitz and Stegun (1965).

Tables of several of these intervals are available in standard statistical references (Natrella, 1966). The factor for the confidence interval for the mean is usually not tabulated because of the ease with which it can be estimated given a table of Student-t statistics. The Tolerance Limit is also frequently tabulated for two sided Confidence Limits but statistical literature must be searched to find tables of one sided Tolerance Limits. A table to generate two-sided Prediction Intervals can be found in Wadsworth (1990).

The published interval tables, however, do not usually apply to numbers of tests, numbers of replicates or numbers of runs that are typical of stack testing. The published tables also ignore the problem of run durations and apply to uninteresting numbers of future tests and runs. Finally, the published tables are usually for two sided intervals and the usual emissions estimating problem involves finding a limit that is not likely to be exceeded (one-sided limit).

The following equations provide the means of calculating the factors applicable to specific problems:

Upper Confidence Interval Limit for the Mean

$$g_{CI} = (1/n)^{1/2} * t_{(n-1,\ 1-alpha)} \tag{B-1}$$

Upper Prediction Interval Limit for k Future Runs

$$g_{PIk} = (1 + 1/n)^{1/2} * t_{(n-1,\ 1-alpha/k)} \tag{B-2}$$

Upper Prediction Interval Limit for the Next Test Mean

$$g_{PIm} = (1/R + 1/n)^{1/2} * t_{(n-1,\ 1-alpha)} \tag{B-3}$$

Upper Prediction Interval for T Future Replicate Means

$$g_{PIRT} = (1/R + 1/n)^{1/2} * t_{(n-1,\ 1-alpha/[T*R])} \tag{B-4}$$

Upper Tolerance Limit

$$g_T = [Z_p + (Z_p^2 - a * b)^{1/2}]/a \tag{B-5}$$

where

$a = 1 - [Z_{(1-alpha)}]^2/[2 * n - 2]$

$b = Zp^2 - Z_{(1-alpha)}{}^2/n$

Upper Confidence Limit on the Standard Deviation

$$g_{SD} = \{(n - 1)/X^2_{(n-1,\ alpha)}\}^{1/2} \tag{B-6}$$

and the factor to correct for R replicates or test duration in-house is g_{SD} divided by $R^{1/2}$ or $R^{0.4}$ (depending on whether the correction is for random averages or block averages) to convert the standard deviation to a one hour result.

In order to use the above formulas, estimates of the Student t-distribution for significance levels (values of alpha) which are not found in common tables are needed. Also, many cumulative normal probability tables do not contain Z_p of interest and interpolation is necessary.

A rational approximation for Z_p given the fraction of the normal curve below p (%f_i expressed as a fraction) such that $0 < p \leq 0.5$ is:

$$Z_p = t - (2.30753 + 0.27061t)/(1 + 0.99229t + 0.04481t^2) \tag{B-7}$$

where:

$t = \{-2.0 * \ln(p)\}^{1/2}$

For p greater than 0.5, compute the above for:

$p' = 1 - p$ and

$Z_p = -Z_p'$

An asymptotic expansion for the inverse Student-t distribution function, which uses Z_p corresponding to alpha, is:

$$t_p = Z_p + G_1(Z_p)/(n-1) + G_2(Z_p)/(n-1)^2 + G_3(Z_p)/(n-1)^3 + G_4(Z_p)/(n-1)^4 \tag{B-8}$$

where:

$G_1(z) = (z^3 + z)/4$

$G_2(z) = (5z^5 + 16z^3 + 3z)/96$

$G_3(z) = (3z^7 + 19z^5 + 17z^3 - 15z)/384$

$$G_4(z) = (79z^9 + 776z^7 + 1482z^5 - 1920z^3 - 945z)/92160$$

Numerical approximations for the Chi-Square distribution are only available for more than 30 degrees of freedom. Since one is usually interested in tabulated confidence levels (alpha levels of 5%, 2.5%, 1% and 0.1%), a need to generate untabulated values is unlikely. Appropriate values can be found in standard statistical references (Natrella, 1966).

APPENDIX C

APPENDIX C-1 FACTORS FOR DETERMINING UPPER CONFIDENCE, TOLERANCE AND PREDICTION LIMITS AT THE 5% SIGNIFICANCE LEVEL (95% Confidence) AND 95% PROBABILITY OF CONTAINING ALL FUTURE RESULTS

DATA BASE	FUTURE TESTS	FUTURE REPS.	FUTURE RUNS	PROBAB-ILITY	CONFID-ENCE	95% CONFID-ENCE LIMIT	95% TOLERANCE LIMIT	PREDICTION LIMITS: NEXT TEST	PREDICTION LIMITS: MEAN OF T FUTURE TESTS	PREDICTION LIMITS: k FUTURE TESTS
n	T	R	k	P	alpha	g CI	g T	g PIm	g PIRT	g PIk
3	1	3	3	95%	5%	1.68	9.57	2.38	4.31	6.09
	2		6						6.09	8.62
	3		9						7.41	10.47
	4		12						8.48	12.00
	5		15						9.41	13.31
	20		60						17.40	24.60
6	1	3	3	95%	5%	0.82	3.67	1.42	2.06	3.15
	2		6						2.50	3.82
	3		9						2.78	4.24
	4		12						2.98	4.56
	5		15						3.15	4.81
	20		60						4.33	6.62
9	1	3	3	95%	5%	0.62	2.99	1.24	1.71	2.71
	2		6						2.01	3.18
	3		9						2.19	3.47
	4		12						2.32	3.67
	5		15						2.42	3.83
	20		60						3.10	4.90
12	1	3	3	95%	5%	0.52	2.71	1.16	1.57	2.53
	2		6						1.82	2.94
	3		9						1.97	3.17
	4		12						2.07	3.34
	5		15						2.15	3.47
	20		60						2.67	4.31
15	1	3	3	95%	5%	0.45	2.54	1.11	1.49	2.44
	2		6						1.72	2.81
	3		9						1.85	3.02
	4		12						1.94	3.17
	5		15						2.01	3.29
	20		60						2.46	4.01
18	1	3	3	95%	5%	0.41	2.43	1.08	1.44	2.38
	2		6						1.66	2.73
	3		9						1.78	2.93
	4		12						1.86	3.07
	5		15						1.93	3.18
	20		60						2.33	3.84
21	1	3	3	95%	5%	0.38	2.35	1.06	1.41	2.34
	2		6						1.61	2.68
	3		9						1.73	2.87
	4		12						1.81	3.00
	5		15						1.87	3.10
	20		60						2.24	3.72

APPENDIX C-2 FACTORS FOR DETERMINING UPPER CONFIDENCE, TOLERANCE AND PREDICTION LIMITS AT THE 1% SIGNIFICANCE LEVEL (99% Confidence) AND 99% PROBABILITY OF CONTAINING ALL FUTURE RESULTS

DATA BASE	FUTURE TESTS	FUTURE REPS.	FUTURE RUNS	PROBABILITY	CONFIDENCE	99% CONFIDENCE LIMIT	99% TOLERANCE LIMIT	PREDICTION LIMITS NEXT TEST	PREDICTION LIMITS MEAN OF T FUTURE TESTS	PREDICTION LIMITS k FUTURE TESTS
n	T	R	k	P	alpha	g CI	g T	g PIm	g PIRT	g PIk
3	1	3	3	99%	1%	3.94	13.87	5.57	9.41	13.31
	2		6						12.88	18.21
	3		9						15.38	21.75
	4		12						17.40	24.60
	5		15						19.12	27.04
	20		60						33.39	47.22
6	1	3	3	99%	1%	1.37	9.07	2.38	3.15	4.81
	2		6						3.71	5.67
	3		9						4.06	6.21
	4		12						4.33	6.62
	5		15						4.55	6.95
	20		60						6.10	9.32
9	1	3	3	99%	1%	0.97	5.78	1.93	2.42	3.83
	2		6						2.75	4.35
	3		9						2.95	4.67
	4		12						3.10	4.90
	5		15						3.21	5.08
	20		60						3.98	6.30
12	1	3	3	99%	1%	0.79	4.81	1.76	2.15	3.47
	2		6						2.41	3.88
	3		9						2.56	4.13
	4		12						2.67	4.31
	5		15						2.76	4.44
	20		60						3.31	5.33
15	1	3	3	99%	1%	0.68	4.32	1.66	2.01	3.29
	2		6						2.23	3.65
	3		9						2.36	3.86
	4		12						2.46	4.01
	5		15						2.53	4.13
	20		60						2.98	4.87
18	1	3	3	99%	1%	0.61	4.03	1.60	1.93	3.18
	2		6						2.13	3.51
	3		9						2.25	3.70
	4		12						2.33	3.84
	5		15						2.39	3.94
	20		60						2.79	4.60
21	1	3	3	99%	1%	0.55	3.82	1.56	1.87	3.10
	2		6						2.06	3.41
	3		9						2.17	3.59
	4		12						2.24	3.72
	5		15						2.30	3.82
	20		60						2.67	4.43

APPENDIX C-3 FACTORS FOR DETERMINING THE STANDARD DEVIATION APPLICABLE TO ONE BLOCK AVERAGING TIME BASIS GIVEN ANOTHER

FACTORS TO CONVERT SAMPLE PERIOD STANDARD DEVIATIONS TO EQUIVALENT ONE HOUR STANDARD DEVIATIONS AT THE 95% CONFIDENCE LEVEL BLOCK AVERAGES

NUMBER OF RUNS IN DATA BASE	SAMPLE RUN TIMES 2	3	4	8	24
3	5.90	6.94	7.79	10.27	15.94
4	3.86	4.54	5.10	6.73	10.44
5	3.13	3.68	4.13	5.45	8.46
6	2.75	3.24	3.63	4.79	7.43
7	2.52	2.97	3.33	4.39	6.82
8	2.37	2.79	3.13	4.13	6.40
9	2.26	2.66	2.98	3.93	6.10
10	2.17	2.55	2.86	3.78	5.86
11	2.10	2.47	2.77	3.66	5.68
12	2.05	2.41	2.70	3.56	5.53
13	2.00	2.35	2.64	3.48	5.40
14	1.96	2.31	2.59	3.41	5.30
15	1.93	2.27	2.54	3.35	5.20
16	1.90	2.23	2.50	3.30	5.12
17	1.87	2.20	2.47	3.26	5.05
18	1.85	2.17	2.44	3.22	4.99
19	1.83	2.15	2.41	3.18	4.94
20	1.81	2.13	2.39	3.15	4.89

FACTORS TO CONVERT ONE HOUR STANDARD DEVIATIONS TO EQUIVALENT AVERAGING PERIOD STANDARD DEVIATIONS AT THE 95% CONFIDENCE LEVEL BLOCK AVERAGES

NUMBER OF RUNS IN DATA BASE	SAMPLE RUN TIMES 2	3	4	8	24
3	3.27	2.73	2.40	1.75	1.07
4	2.14	1.79	1.57	1.15	0.70
5	1.74	1.45	1.27	0.93	0.57
6	1.53	1.27	1.12	0.82	0.50
7	1.40	1.17	1.03	0.75	0.46
8	1.31	1.10	0.96	0.70	0.43
9	1.25	1.04	0.92	0.67	0.41
10	1.20	1.00	0.88	0.64	0.39
11	1.17	0.97	0.85	0.62	0.38
12	1.14	0.95	0.83	0.61	0.37
13	1.11	0.92	0.81	0.59	0.36
14	1.09	0.91	0.80	0.58	0.36
15	1.07	0.89	0.78	0.57	0.35
16	1.05	0.88	0.77	0.56	0.34
17	1.04	0.86	0.76	0.56	0.34
18	1.03	0.85	0.75	0.55	0.34
19	1.01	0.84	0.74	0.54	0.33
20	1.00	0.84	0.73	0.54	0.33

APPENDIX C-4 FACTORS FOR DETERMINING THE STANDARD DEVIATION APPLICABLE TO ONE RANDOM SAMPLE AVERAGING TIME BASIS GIVEN ANOTHER

FACTORS TO CONVERT SAMPLE PERIOD STANDARD DEVIATIONS TO EQUIVALENT ONE HOUR STANDARD DEVIATIONS AT THE 95% CONFIDENCE LEVEL RANDOM AVERAGES

NUMBER OF RUNS IN DATA BASE	SAMPLE RUN TIMES 2	3	4	8	24
3	6.32	7.75	8.94	12.65	21.91
4	4.14	5.07	5.86	8.28	14.34
5	3.36	4.11	4.75	6.71	11.63
6	2.95	3.61	4.17	5.90	10.22
7	2.71	3.31	3.83	5.41	9.37
8	2.54	3.11	3.59	5.08	8.80
9	2.42	2.96	3.42	4.84	8.39
10	2.32	2.85	3.29	4.65	8.05
11	2.25	2.76	3.19	4.51	7.80
12	2.19	2.69	3.10	4.39	7.60
13	2.14	2.62	3.03	4.28	7.42
14	2.10	2.57	2.97	4.20	7.28
15	2.06	2.53	2.92	4.13	7.15
16	2.03	2.49	2.87	4.07	7.04
17	2.01	2.46	2.84	4.01	6.95
18	1.98	2.43	2.80	3.96	6.86
19	1.96	2.40	2.77	3.92	6.78
20	1.94	2.37	2.74	3.88	6.71

FACTORS TO CONVERT ONE HOUR STANDARD DEVIATIONS TO EQUIVALENT AVERAGING PERIOD STANDARD DEVIATIONS AT THE 95% CONFIDENCE LEVEL RANDOM AVERAGES

NUMBER OF RUNS IN DATA BASE	SAMPLE RUN TIMES 2	3	4	8	24
3	3.16	2.58	2.24	1.58	0.91
4	2.07	1.69	1.46	1.04	0.60
5	1.68	1.37	1.19	0.84	0.48
6	1.47	1.20	1.04	0.74	0.43
7	1.35	1.10	0.96	0.68	0.39
8	1.27	1.04	0.90	0.64	0.37
9	1.21	0.99	0.86	0.61	0.35
10	1.16	0.95	0.82	0.58	0.34
11	1.13	0.92	0.80	0.56	0.33
12	1.10	0.90	0.78	0.55	0.32
13	1.07	0.87	0.76	0.54	0.31
14	1.05	0.86	0.74	0.53	0.30
15	1.03	0.84	0.73	0.52	0.30
16	1.02	0.83	0.72	0.51	0.29
17	1.00	0.82	0.71	0.50	0.29
18	0.99	0.81	0.70	0.50	0.29
19	0.98	0.80	0.69	0.49	0.28
20	0.97	0.79	0.69	0.48	0.28

APPENDIX D

APPENDIX D-1 FACTORS AND CALCULATION OF UPPER TOLERANCE LIMITS CORRESPONDING TO ONE EXCEEDANCE PER CALENDAR PERIOD — 1-hr BLOCK AVERAGE DATA

ONE HOUR AVERAGING TIME							
exceedance frequency	mean log normal	std dev distribution	n	P	statistical confidence	UTL	limit PPMdv
daily	4.243	0.192	734	0.974227	0.95	2.054	103
weekly	4.243	0.192	734	0.996285	0.95	2.814	119
monthly	4.243	0.192	734	0.999144	0.95	3.292	131
yearly	4.243	0.192	734	0.999929	0.95	3.988	150
5 years	4.243	0.192	734	0.999986	0.95	4.386	162
10 years	4.243	0.192	734	0.999993	0.95	4.547	167
15 years	4.243	0.192	734	0.999995	0.95	4.639	170
20 years	4.243	0.192	734	0.999996	0.95	4.703	172
	normal	distribution					
daily	70.915	15.276	734	0.974227	0.95	2.054	102
weekly	70.915	15.276	734	0.996285	0.95	2.814	114
monthly	70.915	15.276	734	0.999144	0.95	3.292	121
yearly	70.915	15.276	734	0.999929	0.95	3.988	132
5 years	70.915	15.276	734	0.999986	0.95	4.386	138
10 years	70.915	15.276	734	0.999993	0.95	4.547	140
15 years	70.915	15.276	734	0.999995	0.95	4.639	142
20 years	70.915	15.276	734	0.999996	0.95	4.703	143

exceedance frequency	mean log normal	std dev distribution	n	P	statistical confidence	UTL	limit
daily	4.243	0.192	734	0.974227	0.99	2.101	104
weekly	4.243	0.192	734	0.996285	0.99	2.874	121
monthly	4.243	0.192	734	0.999144	0.99	3.360	133
yearly	4.243	0.192	734	0.999929	0.99	4.068	152
5 years	4.243	0.192	734	0.999986	0.99	4.473	164
10 years	4.243	0.192	734	0.999993	0.99	4.637	170
15 years	4.243	0.192	734	0.999995	0.99	4.731	173
20 years	4.243	0.192	734	0.999996	0.99	4.796	175
	normal	distribution					
daily	70.915	15.276	734	0.974227	0.99	2.101	103
weekly	70.915	15.276	734	0.996285	0.99	2.874	115
monthly	70.915	15.276	734	0.999144	0.99	3.360	122
yearly	70.915	15.276	734	0.999929	0.99	4.068	133
5 years	70.915	15.276	734	0.999986	0.99	4.473	139
10 years	70.915	15.276	734	0.999993	0.99	4.637	142
15 years	70.915	15.276	734	0.999995	0.99	4.731	143
20 years	70.915	15.276	734	0.999996	0.99	4.796	144

APPENDIX D-2 FACTORS AND CALCULATION OF UPPER TOLERANCE LIMITS CORRESPONDING TO ONE EXCEEDANCE PER CALENDAR PERIOD — 4-hr BLOCK AVERAGE DATA

FOUR HOUR AVERAGING TIME

exceedance frequency	mean log normal	std dev distribution	n	P	statistical confidence	UTL	limit PPMdv
daily	4.243	0.110	734	0.900000	0.95	1.365	81
weekly	4.243	0.110	734	0.985207	0.95	2.292	90
monthly	4.243	0.110	734	0.996580	0.95	2.843	95
yearly	4.243	0.110	734	0.999715	0.95	3.615	104
5 years	4.243	0.110	734	0.999943	0.95	4.045	109
10 years	4.243	0.110	734	0.999971	0.95	4.218	111
15 years	4.243	0.110	734	0.999981	0.95	4.317	112
20 years	4.243	0.110	734	0.999986	0.95	4.386	113
	normal distribution						
daily	70.915	8.774	734	0.900000	0.95	1.365	83
weekly	70.915	8.774	734	0.985207	0.95	2.292	91
monthly	70.915	8.774	734	0.996580	0.95	2.843	96
yearly	70.915	8.774	734	0.999715	0.95	3.615	103
5 years	70.915	8.774	734	0.999943	0.95	4.045	106
10 years	70.915	8.774	734	0.999971	0.95	4.218	108
15 years	70.915	8.774	734	0.999981	0.95	4.317	109
20 years	70.915	8.774	734	0.999986	0.95	4.386	109

exceedance frequency	mean log normal	std dev distribution	n	P	statistical confidence	UTL	limit
daily	4.243	0.192	734	0.900000	0.99	1.401	91
weekly	4.243	0.192	734	0.985207	0.99	2.343	109
monthly	4.243	0.192	734	0.996580	0.99	2.903	122
yearly	4.243	0.192	734	0.999715	0.99	3.688	141
5 years	4.243	0.192	734	0.999943	0.99	4.126	154
10 years	4.243	0.192	734	0.999971	0.99	4.303	159
15 years	4.243	0.192	734	0.999981	0.99	4.403	162
20 years	4.243	0.192	734	0.999986	0.99	4.473	164
	normal distribution						
daily	70.915	15.276	734	0.900000	0.99	1.401	92
weekly	70.915	15.276	734	0.985207	0.99	2.343	107
monthly	70.915	15.276	734	0.996580	0.99	2.903	115
yearly	70.915	15.276	734	0.999715	0.99	3.688	127
5 years	70.915	15.276	734	0.999943	0.99	4.126	134
10 years	70.915	15.276	734	0.999971	0.99	4.303	137
15 years	70.915	15.276	734	0.999981	0.99	4.403	138
20 years	70.915	15.276	734	0.999986	0.99	4.473	139

APPENDIX D-3 FACTORS AND CALCULATION OF UPPER TOLERANCE LIMITS CORRESPONDING TO ONE EXCEEDANCE PER CALENDAR PERIOD — 24-hr BLOCK AVERAGE DATA

TWENTY FOUR HOUR AVERAGING TIME							
exceedance frequency	mean log normal	std dev distribution	n	P	statistical confidence	UTL	limit PPMdv
daily	4.243	0.054	734	0.500000	0.95	0.061	70
weekly	4.243	0.054	734	0.913793	0.95	1.450	75
monthly	4.243	0.054	734	0.979620	0.95	2.157	78
yearly	4.243	0.054	734	0.998289	0.95	3.075	82
5 years	4.243	0.054	734	0.999658	0.95	3.563	84
10 years	4.243	0.054	734	0.999829	0.95	3.756	85
15 years	4.243	0.054	734	0.999886	0.95	3.865	86
20 years	4.243	0.054	734	0.999914	0.95	3.941	86
	normal	distribution					
daily	70.915	4.285	734	0.500000	0.95	0.061	71
weekly	70.915	4.285	734	0.913793	0.95	1.450	77
monthly	70.915	4.285	734	0.979620	0.95	2.157	80
yearly	70.915	4.285	734	0.998289	0.95	3.075	84
5 years	70.915	4.285	734	0.999658	0.95	3.563	86
10 years	70.915	4.285	734	0.999829	0.95	3.756	87
15 years	70.915	4.285	734	0.999886	0.95	3.865	87
20 years	70.915	4.285	734	0.999914	0.95	3.941	88

exceedance frequency	mean log normal	std dev distribution	n	P	statistical confidence	UTL	limit
daily	4.243	0.192	734	0.500000	0.99	0.086	71
weekly	4.243	0.192	734	0.913793	0.99	1.488	93
monthly	4.243	0.192	734	0.979620	0.99	2.206	106
yearly	4.243	0.192	734	0.998289	0.99	3.138	127
5 years	4.243	0.192	734	0.999658	0.99	3.635	140
10 years	4.243	0.192	734	0.999829	0.99	3.832	145
15 years	4.243	0.192	734	0.999886	0.99	3.943	148
20 years	4.243	0.192	734	0.999914	0.99	4.020	151
	normal	distribution					
daily	70.915	15.276	734	0.500000	0.99	0.086	72
weekly	70.915	15.276	734	0.913793	0.99	1.488	94
monthly	70.915	15.276	734	0.979620	0.99	2.206	105
yearly	70.915	15.276	734	0.998289	0.99	3.138	119
5 years	70.915	15.276	734	0.999658	0.99	3.635	126
10 years	70.915	15.276	734	0.999829	0.99	3.832	129
15 years	70.915	15.276	734	0.999886	0.99	3.943	131
20 years	70.915	15.276	734	0.999914	0.99	4.020	132