POWER GENERATION ALTERNATIVES—SMALL-SCALE WASTE-TO-ENERGY SYSTEMS

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ABSTRACT

As landfill-based waste disposal options become more costly, or their use restricted, an ever increasing number of smaller-sized resource recovery plants [i.e., less than 500 TPD (450 tpd)] will be evaluating alternatives for production of electricity. This would include the use of backpressure turbines and extraction condensing units in cogeneration applications and condensing turbines in all-electric plants. Power production concepts, equipment arrangements and configurations, potential electricity generating rates, and steam quality effects on turbine generator set performance will be discussed. The paper will highlight typical efficiency levels and systems' anticipated gross and net production levels. Detailed data from existing plants and those under construction will be drawn upon to overview vendor activity and facility costs for these small electricity generating plants. Typical ranges of vendor guarantees at 100 TPD (90 tpd) to 500 TPD (450 tpd) will be presented.

INTRODUCTION

The steam produced in a solid waste-to-energy plant boiler may be converted into electrical energy in a turbine generator. This paper examines the operating parameters associated with small turbine generators as applied to various types and sizes of small-scale waste-to-energy recovery systems. The present manufacturers (and their equipment size ranges) and representative capital costs of this equipment are also presented.

Cogeneration can reduce the waste-to-energy plant's tipping fee vulnerability by providing: (1) improved system efficiency; (2) greater control on the energy by-products; and (3), in some instances, greater control of the overall system reliability. Cogenerators can operate in a number of different energy sales modes, including "buy all/sell all" and "excess" electricity sales as well as sale of steam to nearby steam users. In a buy-all/sell-all mode, all the electricity is sold to the grid and the needs of the plant are met by purchased electricity. Under this mode, the cogenerated electricity may be sold to the utility at a price higher than that paid for purchased power (obviously, this is a very utility-specific situation). In an excess sale method, the gross electricity generated is used to reduce the plant's demand and energy charges with any surplus or "excess" power sold to the grid.

TECHNICAL

Introduction to Power Production

Steam turbines have been developed for the generation of electrical power. The steam turbine is used to produce rotary motion or usable shaft power. To achieve moderately good turbine efficiency, the input of high pressure, superheated steam while exhausting at subatmospheric pressures is required. This particular scheme, typically accomplished using a condensing turbine,[1] provides maximum power output per steam flow input. Depending upon the specific thermal needs of the project, consideration may be given to the use of noncondensing turbines[1] as well. However, regardless of the style of turbine selected, the shaft power generated is used to "turn" a generator and thereby produce electricity. The following sections describe the use of both condensing and noncondensing turbines for power production.

Steam Turbines for Electrical Production Only

A diagram of a conventional waste-to-energy steam plant is provided in Fig. 1. This simplified arrangement reflects only a single energy product plant (i.e., only steam production). For power production, a steam turbine would be introduced to the system after the boiler. The pressure differential between the steam input and the exhaust conditions of the turbine is an important indicator of the relative degree of power production. As steam enters a turbine (see Fig. 2), it is expanded through a stationary nozzle, thereby transforming the steam's heat energy to velocity energy. The expanded steam jet is allowed to strike the blades on the turbine wheel thereby making the wheel revolve.[2] Depending upon the specific design conditions (steam flow, pressure, temperature, etc.), turbines may be either single stage or multi-staged, with the latter being the more efficient. This greater efficiency is accomplished at significantly higher capital costs that must be offset by higher system revenues.

Since the potential capital cost of small scale waste-to-electricity projects could range from $8 to 10 million [100 TPD (90 tpd plant)] up to $40 to 50 million [500 TPD (450 tpd plant)], the importance of steam quality (pressure/temperature) on the turbine performance for power-only plant outputs needs to be carefully investigated. The energy input to a steam turbine is supplied by a steam generator, or boiler, which should be designed to produce steam at pressures of at least 50 psig (344 kPa) greater than the actual operating pressure introduced to the turbine inlet. The enthalpy (h) of steam, measured in Btu/lb (MJ/kg), depends on and changes with the actual steam pressure and temperature generated. (Note: The enthalpy of steam can be found on charts in thermodynamic books and other technical references with steam tables.) The maximum portion of the steam's potential energy that can be converted to shaft horsepower is defined as the theoretical steam rate (TSR). [Note: The American Society of Mechanical Engineers' (ASME) books are also available on the TSR's of steam at various inlet and outlet conditions.] Table 1 reflects, for illustrative purposes, the difference of turbine inlet steam pressure and temperature conditions on the TSR.

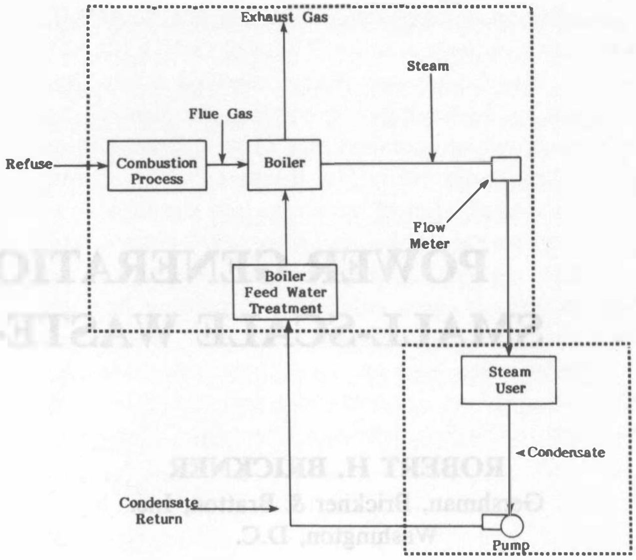

FIG. 1 BASIC COMPONENTS OF A WASTE-TO-ENERGY SYSTEM WITH STEAM PRODUCTION ONLY

As can be calculated by the use of Table 1, if no inefficiencies were assumed for the mechanical devices, a steam flow of 20,000 lb/hr at steam conditions of 250 psig (1720 kPa), 406°F (208°C), with a 3 in. Hg Abs. exhaust condition would be able to theoretically generate 1957 kW of electricity (i.e., 20,000 lb/hr ÷ 10.22 kg/kW·h). However, since multi-stage turbine-generators are nominally 55–60% efficient at this size range (see Table 2), the actual (versus theoretical) output would be expected to approximate 1100 kW for this size system. As can be seen in Table 1, both a temperature increase at a constant pressure or pressure and temperature increases at the same steam flow rate

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[1] A condensing turbine uses the energy contained in steam for power generation only, whereas a noncondensing turbine exhausts steam at the back end. This steam can be used for process energy or space heating.

[2] Impulse turbines do not have a pressure drop across the rotating blades. Velocity impingement on the rotating bladder causes the wheel to rotate.

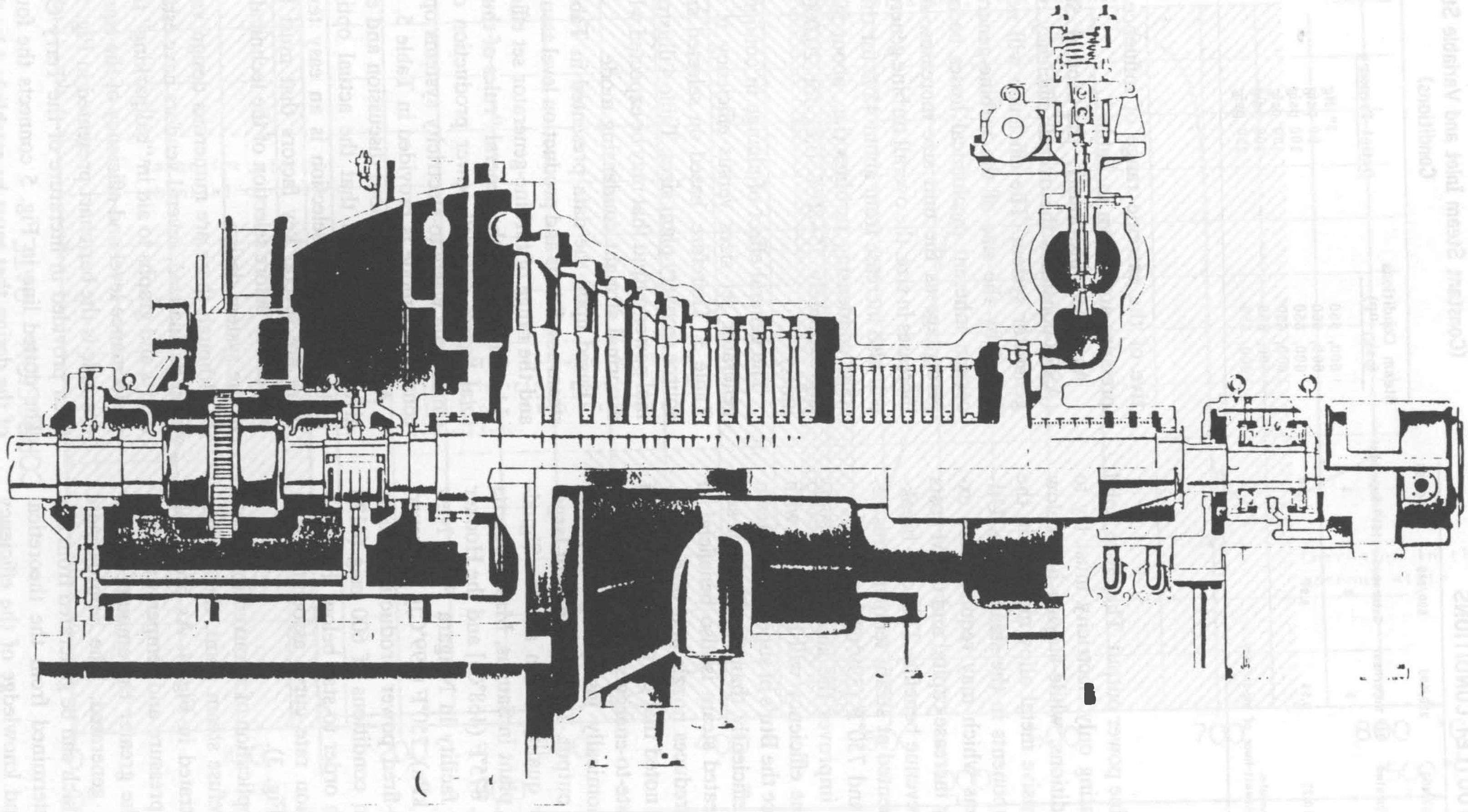

FIG. 2 STEAM TURBINE CROSS SECTION
(Courtesy of Turbodyne Division, Dresser Industries, Inc.)

TABLE 1 THEORETICAL STEAMING RATES VS STEAM INLET/OUTLET CONDITIONS

Inlet Pressure, Temperature (psig/°F)	250/406	250/500	600/489	600/650
Steam Condition	Saturated	94° Superheat	Saturated	161° Superheat
Exhaust Pressure (Hg Abs.)	3	3	3	3
TSR (#/kWh)	10.22	9.59	8.97	7.90

Hg Abs. = inches of mercury absolute.
#/kWh = pounds of steam per kilowatt-hour of electricity.

TABLE 2 THEORETICAL STEAM RATES (Constant Steam Inlet and Variable Steam Outlet Conditions)

Steam Conditions (psig, °F)	Outlet Pressure	Theoretical Steam Rate (#/kWh)
600, 650	3" Hg	7.9
600, 650	50 psig	16.9
600, 650	100 psig	21.7
600, 650	150 psig	26.7
600, 650	200 psig	32.5
600, 650	250 psig	39.6

can improve the turbine power output. Therefore, solid waste projects generating only electricity must try to maximize steam conditions, while staying just below requirement for expensive metal alloy material in the boiler and other components in the steam cycle. Utilizing steam conditions which may require more expensive materials may increase capital and O&M costs above the increased revenue benefits. This cost breakpoint is typically presented at steam temperatures between 650°F (343°C) and 750°F (399°C).

Superheated steam improves the actual "net cycle efficiency." The turbine efficiency will be higher with superheated steam since the Btu's of superheated steam can be utilized more efficiently than the Btu's of saturated steam. Superheated steam is also beneficial to a steam turbine as it reduces nozzle and blade wear. However, it should be noted that certain economies of scale of the larger waste-to-energy systems (500 TPD or greater) may economically justify higher quality steam for more power output. Two examples of larger plants utilizing higher quality steam are the Signal-RESCO mass burning plant in Saugus, Massachusetts [690 psig (4747.2 kPa), 875°F (468°C)] and the Hooker Chemical RDF boiler facility in Niagara Falls, New York [1250 psig (8600 kPa), 750°F (399°C)]. However, the majority of refuse-fired power production plants are designed for steam conditions of 600 psig (4128 kPa), 650°F (343°C), in order to stay below the accelerated chloride corrosion rate curve associated with the boiler design (see Fig. 3).

In considering the application of a conventional condensing turbine on a refuse steam plant, the configuration may be as illustrated in Fig. 4. As previously noted, the greater the pressure and temperature drop through the turbine, the greater the amount of electricity which can be generated. The approximate amount of electricity which can be generated from any steam flow may be determined from the theoretical steaming rate charts and knowledge of the efficiency of the turbine-generator (T-G) set. Examples illustrative of the extreme range of influence of the outlet pressure are shown in Table 2.

As noted previously, the Actual Steaming Rate (ASR) considers the overall efficiency of the turbine-generator system. The efficiency will normally be affected by the size of the turbine-generator set since certain inherent mechanical losses become a smaller percentage as the unit size increases. As the system increases in size, the overall turbine generator efficiency will also increase from about 45% for the small multistage condensing turbines (i.e., about 500 kW) up to approximately 75–80% (i.e., for 20,000–30,000 kW units).

The general effect of change in condensing turbine-generator set sizes versus efficiency is illustrated in Table 3. Therefore, based on selected small-scale resource recovery plant sizes, Table 4 illustrates the nominal power output that may be expected when operating a normal straight condensing mode.

Based upon the data presented in Table 4 and the steam conditions and production level assumed therein, and the range of turbine-generator set efficiencies provided in Table 3, the general "rules of thumb" for the total "gross output" power production capability in small-scale waste-to-electricity systems operating in a condensing mode is provided in Table 5.

Although the previous discussion and exhibits may give the impression that the actual optimization of turbine-generator selection is an easy textbook feat, there are several key factors that must be carefully considered before selection of the technical parameters of the system design.

Although there are numerous design variables that must be finalized, several vendors have established bar charts and graphs to aid in "ballparking" the potential performance level and efficiency of the equipment. An example is the bar chart presented in Fig. 5 which has been provided in literature of the Terry Corporation. The dotted line in Fig. 5 connects the four elements of the design that must be established by the designer for proper turbine selection. Starting at the upper left

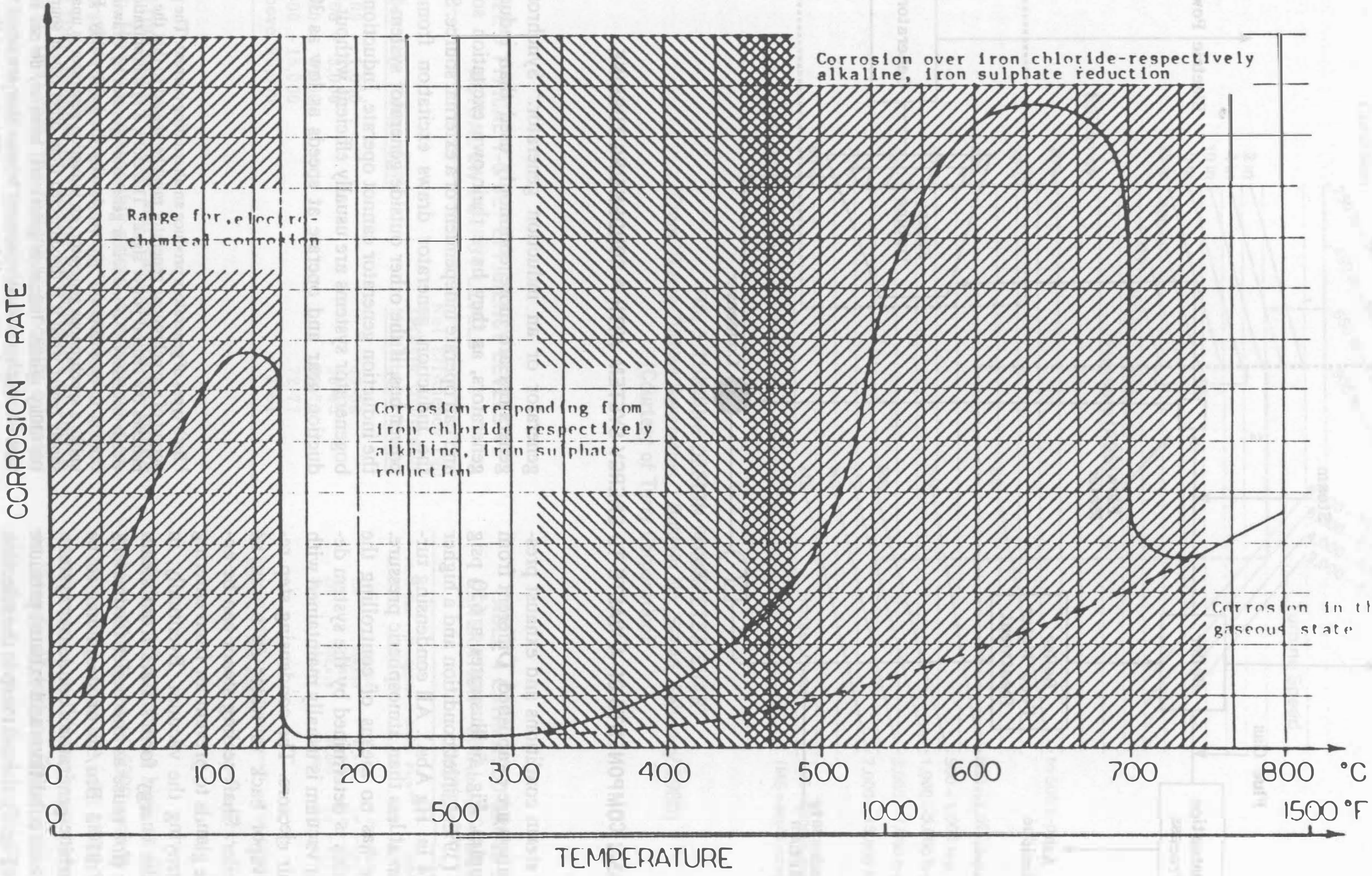

FIG. 3 TEMPERATURE CORROSION GRAPH
(From U.S. Department of Energy. *Proceedings of the International Conference on European Waste-to-Energy Technology.* Argonne National Laboratory, September 1980.)

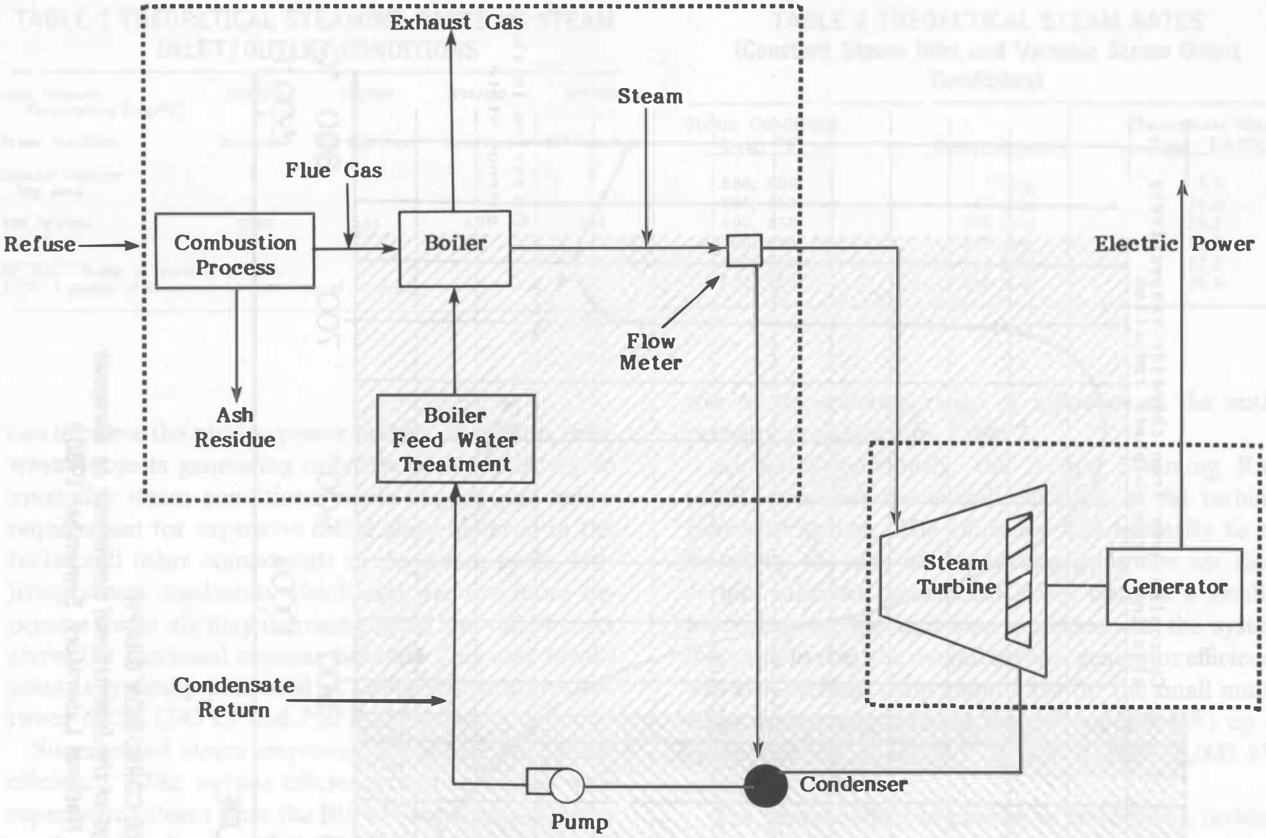

FIG. 4 BASIC COMPONENTS OF A WASTE-TO-ENERGY SYSTEM WITH A CONDENSING TURBINE

hand corner, inlet steam conditions and exhaust pressure for the application are established. Different from the previous examples, Fig. 5 illustrates a 650 psig (4472 kPa)/750°F (399°C) inlet condition and a higher exhaust pressure (4 in. Hg Abs.). All condensing turbines exhaust steam at less than atmospheric pressure. The steam turbine has no means of controlling the exhaust pressure; this is determined by the system designer. The exhaust vacuum is usually maintained with a condenser and air ejectors. The condensing step reduces the steam vapor back to water so it can be returned to the boiler. Shaft-sealing steam can be applied to the turbine glands to prevent air from entering the seals and destroying the vacuum. Eventually, in order to change the energy form's state from steam back to hot water (for reuse as the boiler feedwater), the latent heat of 970.3 Btu/lb of steam has to be removed by the turbine-generator's condenser system.

After the inlet steam conditions and exhaust pressure are established (see Fig. 5), the next step is the selection of the turbine speed. Part of the speed issue is dependent upon whether the system is to drive a synchronous generator or an induction generator.[3] Synchronous generators are more commonly used than induction generators, as they have their own excitation source and can operate independent of a external source. Since the induction generator draws excitation from the power bus, if the other outside generator system fails, the induction generator cannot operate. Induction turbogenerator systems are usually efficient without a reduction gear and operate at speeds as low as 3000–

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[3] Both types of generators produce an alternating current. The power output of a synchronous generator must be phased so that the signal matches the central power signal. Therefore, a synchronization device is necessary. An induction generator operates by drawing excitation from an outside source, usually the local utility. Hence, unlike a synchronous generator, an induction generator is unable to supply power when it is disconnected from other power sources in the utility system. Induction generators, however, do not require separate synchronization equipment because they are timed by the utility signal during parallel operation. This is one of the major advantages of induction generators compared to synchronous generators. However, they are less efficient than a synchronous unit.

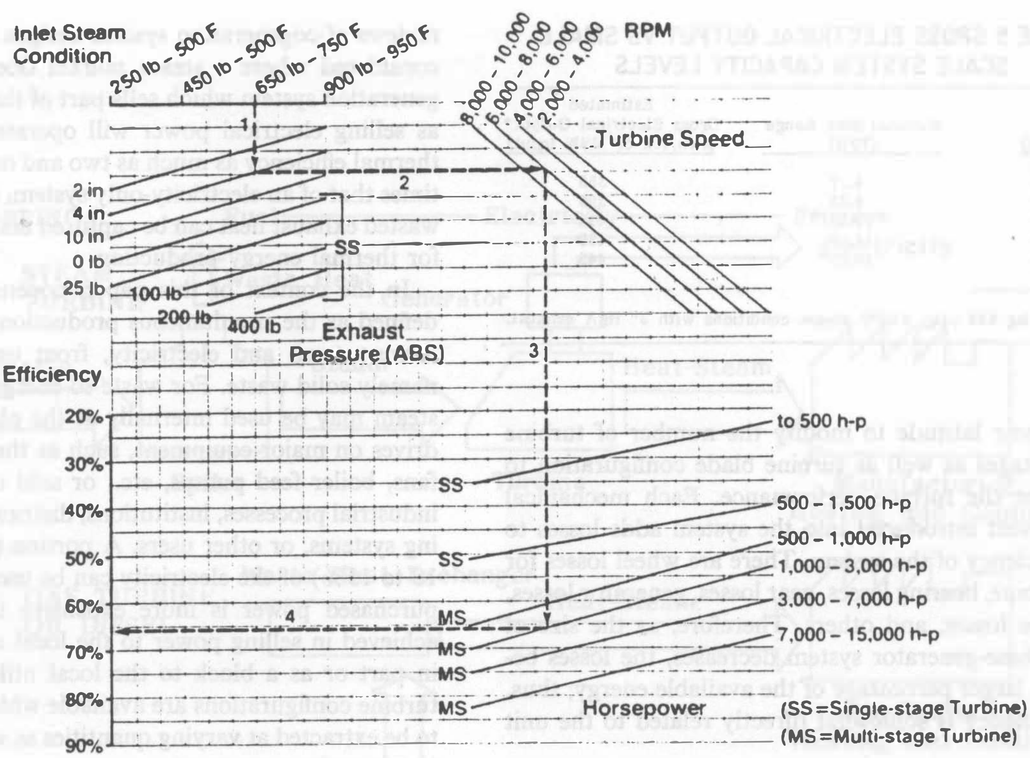

FIG. 5 TURBINE DESIGN PARAMETER CHART
(From "Terry Turbine Generators," Bulletin No. S-246, February 1980)
(Courtesy of Terry Corporation)

TABLE 3 TYPICAL T-G SET EFFICIENCY LEVELS VS ACTUAL kW OUTPUT LEVELS

T-G Set Size Range (kW)	Net Efficiency (%) Turbine Generator Multistage Design
500 - 1,000	45-58
1,000 - 3,000	58-65
3,000 - 7,000	65-72
7,000 - 15,000	72-77
above 20,000	78-81

TABLE 4 ESTIMATED GROSS ELECTRICAL GENERATION—SMALL-SCALE CONDENSING SYSTEMS

Plant Size (TPD)	Steam Conditions (psig, °F)	Steam Flow* (#/hr)	ASR** (#/kWh)	Nominal Output (kW)	(hp)
100	600,650	18,765	7.9 ÷ 0.62 = 12.75	1,470	1,970
200	600,650	40,000	7.9 ÷ 0.65 = 12.15	3,290	4,410
300	600,650	60,000	7.9 ÷ 0.68 = 11.60	5,170	6,930
400	600,650	80,000	7.9 ÷ 0.71 = 11.10	7,200	9,660
500	600,650	100,000	7.9 ÷ 0.73 = 10.80	9,260	12,400

* Based on 4,800# steam/ton processed (except 4,500#/ton at 100 TPD).
** Based on 3" HgA exhaust.

3600 rpm using available and reasonably-priced equipment. However, reasonably-priced synchronous generators for ratings lower than 12,500 kW run at 1500–1800 rpm. Because of turbine inefficiencies at speeds of 1500–1800 rpm, the turbine is generally operated at a higher speed and "geared down" to match the speed requirements of the generator. Most gears are high speed, double-helical, parallel shaft units designed to deliver efficiencies as high as 98.5%. In Fig. 5, a turbine shaft speed of 6000 rpm is assumed.

Depending upon the inlet steam flow quantity and conditions, as well as exhaust pressure, the available or "standard" turbine frame sizes of the vendors will also play a part in final turbine generator efficiency. Without getting into special "engineered systems," the "off-the-shelf" frame sizes provide the vendor designers

TABLE 5 GROSS ELECTRICAL OUTPUT VS SMALL-SCALE SYSTEM CAPACITY LEVELS

(TPD)	Nominal Size Range (TPH)	Estimated Gross Electrical Output* (kwh/ton of MSW Input)
100	4.17	355
200	8.33	395
300	12.50	415
400	16.67	430
500	20.83	445

* Assuming 600 psig, 650°F steam conditions with 3" HgA exhaust.

with some latitude to modify the number of turbine blade stages as well as turbine blade configuration to optimize the turbine performance. Each mechanical component introduced into the system adds losses to the efficiency of the system. There are wheel losses for the turbine, bearing losses, gear losses, generator losses, windage losses, and others. Therefore, as the size of the turbine-generator system decreases, the losses become a larger percentage of the available energy; thus, the efficiency is somewhat directly related to the unit size.

The last selection point guideline in using Fig. 5 is the decision on a multi-stage versus single stage turbine system. For a waste-to-electricity project where 15 or 20 year project life cycles are being evaluated, the use of a multi-stage turbine will almost always be justified, since electricity is the only revenue producer and five to ten percentage points of efficiency gain may be a very cost effective investment. Therefore, almost all straight condensing waste-to-energy systems will employ the multi-stage (MS) steam turbine design. Based upon the T-G set size range, the nominal efficiency can be approximated. As noted earlier, this efficiency is applied as follows: *TSR* ÷ efficiency = *ASR*. The total expected steam flow is divided by the *ASR* to estimate the kW output.

In summary, a straight condensing turbine is used when electrical power generation is the only concern and it must be produced on a minimum amount of steam. To improve the plant's efficiency, these units can be provided with multiple uncontrolled extraction openings for feedwater heating to improve the cycle efficiency (see later section on regenerative cycle).

Turbines for Cogeneration Applications

In addition to operating small-scale waste-to-energy plants as electric-only facilities where steam needs are physically nonexistent or economically not justifiable, reviews of cogeneration system designs should also be considered where a steam market does exist. A cogeneration system which sells part of the steam as well as selling electrical power will operate at an overall thermal efficiency as much as two and one half to three times that of an electricity-only system, if the normally wasted exhaust heat can be captured and partially used for thermal energy production.

In the context of this paper, cogeneration will be defined as the simultaneous production of two energy forms, steam and electricity, from one fuel source, namely solid waste. For waste-to-energy products, the steam may be used internally at the plant for turbine drives on major equipment, such as the induced draft fans, boiler feed pumps, etc., or sold off-site to local industrial processes, institutions, district heating/cooling systems, or other users. A portion (approximately 10 to 15%) of the electricity can be used internally (if purchased power is more expensive than the value achieved in selling power to the local utility), or sold in part or as a block to the local utility. Numerous turbine configurations are available which allow steam to be extracted at varying quantities as well as qualities (pressure/temperature).

Typical congeneration system operating modes are either known as "topping" or "bottoming" cycles as illustrated in Fig. 6. In a topping system, thermal energy exhausted in the production of electrical or mechanical energy is used in industrial processes or for district heating and/or cooling systems. Bottoming-cycle cogeneration reverses this process. Fuel is consumed to produce the high temperature steam needed in an industrial process (such as paper production or aluminum remelting). Heat is extracted from the hot exhaust waste stream and, through a heat exchanger (typically a waste heat recovery boiler), used to drive a turbine to produce electrical or mechanical energy. Typical topping cycle prime movers associated with waste-to-energy plants include the use of extraction turbines and back pressure turbines. Since typical bottoming cycle prime movers include low-pressure turbines and Organic Rankine engines whose waste heat fuel supply is typically of conventional fuels (i.e., gas, oil, coal), topping cycle cogeneration systems are considered as most representative of solid waste-to-energy projects as topping cycles prime movers burn fuel directly.

Since the industrial (or institutional) process steam system is defined as to pressure, temperature, and flow by the steam user (i.e., independent of the waste-to-energy system), the design optimization will primarily focus on the following parameters: the steam pressure from the steam boiler, the boiler feedwater heater and

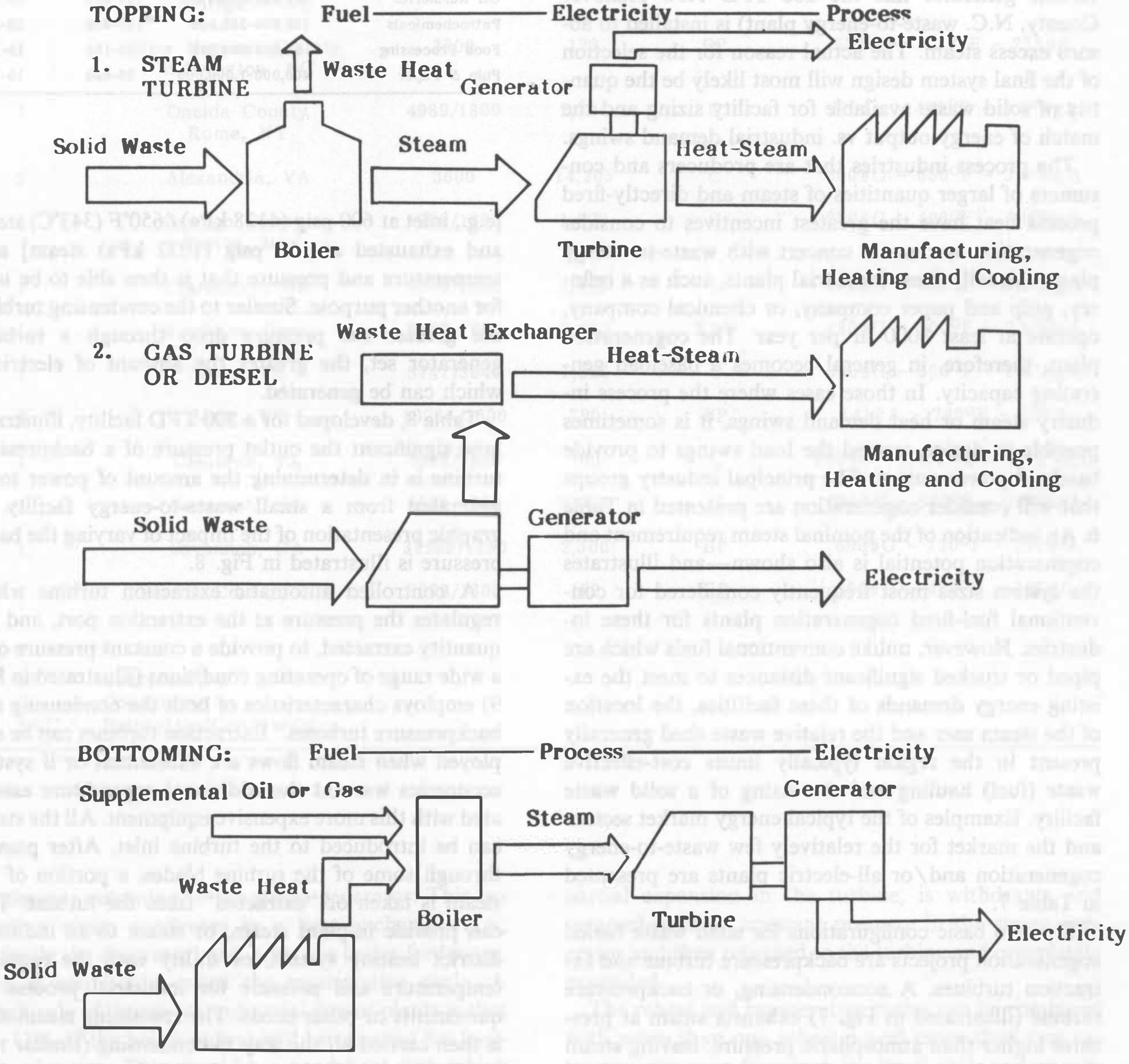

FIG. 6 COGENERATION OPERATING CYCLES
(From "Cogeneration: Energy for the 80s and Beyond." California Governor's Office of Planning and Research, October 1980.)

deaerator system employed, and the number of steam turbines.

In this cycle, a change in steam demand will result in a change in cogenerated power output except if an auxiliary condenser (perhaps with a condensing steam turbine generator like the 200 TPD New Hanover County, N.C. waste-to-energy plant) is installed to absorb excess steam. The actual reason for the selection of the final system design will most likely be the quantity of solid waste available for facility sizing and the match of energy output vs. industrial demand swings.

The process industries that are producers and consumers of larger quantities of steam and directly-fired process heat have the greatest incentives to consider cogeneration systems in concert with waste-to-energy plants. Usually these industrial plants, such as a refinery, pulp and paper company, or chemical company, operate at least 8000 hr per year. The cogeneration plant, therefore, in general becomes a baseload generating capacity. In those cases where the process industry steam or heat demand swings, it is sometimes possible to design around the load swings to provide baseload power output. The principal industry groups that will consider cogeneration are presented in Table 6. An indication of the nominal steam requirement and cogeneration potential is also shown—and illustrates the system sizes most frequently considered for conventional fuel-fired cogeneration plants for these industries. However, unlike conventional fuels which are piped or trucked significant distances to meet the existing energy demands of these facilities, the location of the steam user and the relative waste shed generally present in the region typically limits cost-effective waste (fuel) hauling and the sizing of a solid waste facility. Examples of the typical energy market sectors and the market for the relatively few waste-to-energy cogeneration and/or all-electric plants are presented in Table 7.

The two basic configurations for solid waste fueled cogeneration projects are backpressure turbine and extraction turbines. A noncondensing, or backpressure turbine (illustrated in Fig. 7) exhausts steam at pressures higher than atmospheric pressure, leaving steam that is able to perform other work. The pressure level of the backpressure steam exhaust is normally determined by the specific steam pressure needs of the ultimate steam user. In a backpressure application, the original steam product at the boiler header is of higher quality in terms of temperature and pressure than that which is required for the market and/or internal steam needs. Electrical production is accomplished by passing the higher condition steam through a backpressure turbine that drives a generator. Steam is exhausted [e.g., inlet at 600 psig (4128 kPa)/650°F (343°C) steam and exhausted at 150 psig (1032 kPa) steam] at a temperature and pressure that is then able to be used for another purpose. Similar to the condensing turbine, the greater the pressure drop through a turbine-generator set, the greater the amount of electricity which can be generated.

TABLE 6 EXAMPLES OF THE PRINCIPAL INDUSTRIES WITH COGENERATION POTENTIAL

Industry	Steam(#/hr)	PSIG	MW
Oil Refineries	100,000-1,500,000	150-600	20-300
Petrochemicals	100,000-500,000	150-450	20-100
Food Processing	50,000-500,000	20-150	10-100
Pulp & Paper	400,000-1,000,000	50-600	10-100

Table 8, developed for a 500 TPD facility, illustrates how significant the outlet pressure of a backpressure turbine is in determining the amount of power to be generated from a small waste-to-energy facility. A graphic presentation of the impact of varying the backpressure is illustrated in Fig. 8.

A controlled automatic extraction turbine which regulates the pressure at the extraction port, and the quantity extracted, to provide a constant pressure over a wide range of operating conditions (illustrated in Fig. 9) employs characteristics of both the condensing and backpressure turbines.[4] Extraction turbines can be employed when steam flows are substantial or if system economics warrant the additional expenditure associated with this more expensive equipment. All the steam can be introduced to the turbine inlet. After passing through some of the turbine blades, a portion of the steam is taken or "extracted" from the turbine. This can provide in-plant steam, or steam to an industry, district heating system, or utility with the required temperature and pressure for industrial process requirements or other needs. The remaining steam flow is then carried all the way to condensing (similar to a condensing turbine) to maximize electricity production.

As opposed to the straight condensing T-G set, the efficiency of a steam cycle can be improved by removing some of the steam, after partial expansion in the turbine, for use in increasing the temperature of the

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[4] An uncontrolled extraction port, typically used for feedwater heating, does not control the pressure at the extraction stage.

TABLE 7 REPRESENTATIVE SMALL-SCALE WASTE-TO-ENERGY PROJECTS
(With Electricity Production)

NUMBER OF UNITS	LOCATION	SPEED (rpm)	kW	TYPE OF TURBINE[1]	STM. CONDITIONS
1	New Hanover County Wilmington, NC	5000/1800	2,085	C	555#G - 550°F - 4.0"HgA
1	New Hanover County Wilmington, NC	3600	123	BP	550#G - 550°F - 275#G
1	Oneida County Rome, NY	4989/1800	2,204	C	265#¢ - 411°F - 4.0"HgA
2	Alexandria, VA	3600	14,205	C	600#G - 650°F - 3.0"HgA
2	Oswego County Fulton, NY	5502/1800	1,820	C	250#G - 406°F - 4.0"HgA
1	Claremont, NH	6500/1800	4,485	C	600#G - 600°F - 2.5"HgA
1	Windham, Conn.	5500/1800	2,250	EXC	280#G - 500°F - 2.5"HgA
1	Sealy, TX	5747/3640	1,216	C	300#G - 560°F - 2.0"HgA
1	Sealy, TX	10765/3600	790	BP	141#A - 740°F - 10#A
1	Cleburne, TX	5608/1800	700	C	150#G - 97.4%Q - 12.0"HgA
1	Savannah, GA	4522/1800	2,915	C	275#G - 570°F - 3.0"HgA
1	Savannah, GA	32950/1800	2,500	BP	600#G - 750°F - 275#G
1	Barron County, Wis.	5000/1800	182	BP	500#G - 470°F - 125#G

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1. BP = Backpressure
C = Condensing
EXC = Extraction/Condensing

condensate returning to the steam generator. This extraction steam condenses in a heat exchanger (i.e., typically the deaerator) and heats the boiler feedwater. This is commonly called the regenerative cycle. A rough rule of thumb for industrial power plants is that 10–12% of the boiler output will be required for feedwater heating. This provides a good heat sink which can conveniently be supplied by steam turbine-driven boiler feed pumps. It is possible to install a steam turbine-driven boiler feed pump and use the exhaust steam from the turbine in the deaerator for feedwater heating instead of taking the steam for feedwater heating directly off the boiler through a pressure reducing valve.

Another use of extracted steam is accomplished via the reheat cycle. In the reheat cycle, the steam, after partial expansion in the turbine, is withdrawn and resuperheated at constant pressure in the steam generator and then returned to the turbine and completely expanded.

The reheat and regenerative cycles can be combined with more than one reheater and more than one extraction pressure. However, the small-scale system economics must be carefully reviewed to assure a positive cash flow trend. The regenerative cycle has the advantage of reducing the amount of steam that has to be condensed. This means the lower pressure stages of the turbine do not have to pass as much steam as a straight condensing turbine; and the condenser and circulating water pumps can be smaller.

Figure 10 is an example of the application of an automatic extraction turbine. Part-load steam flow

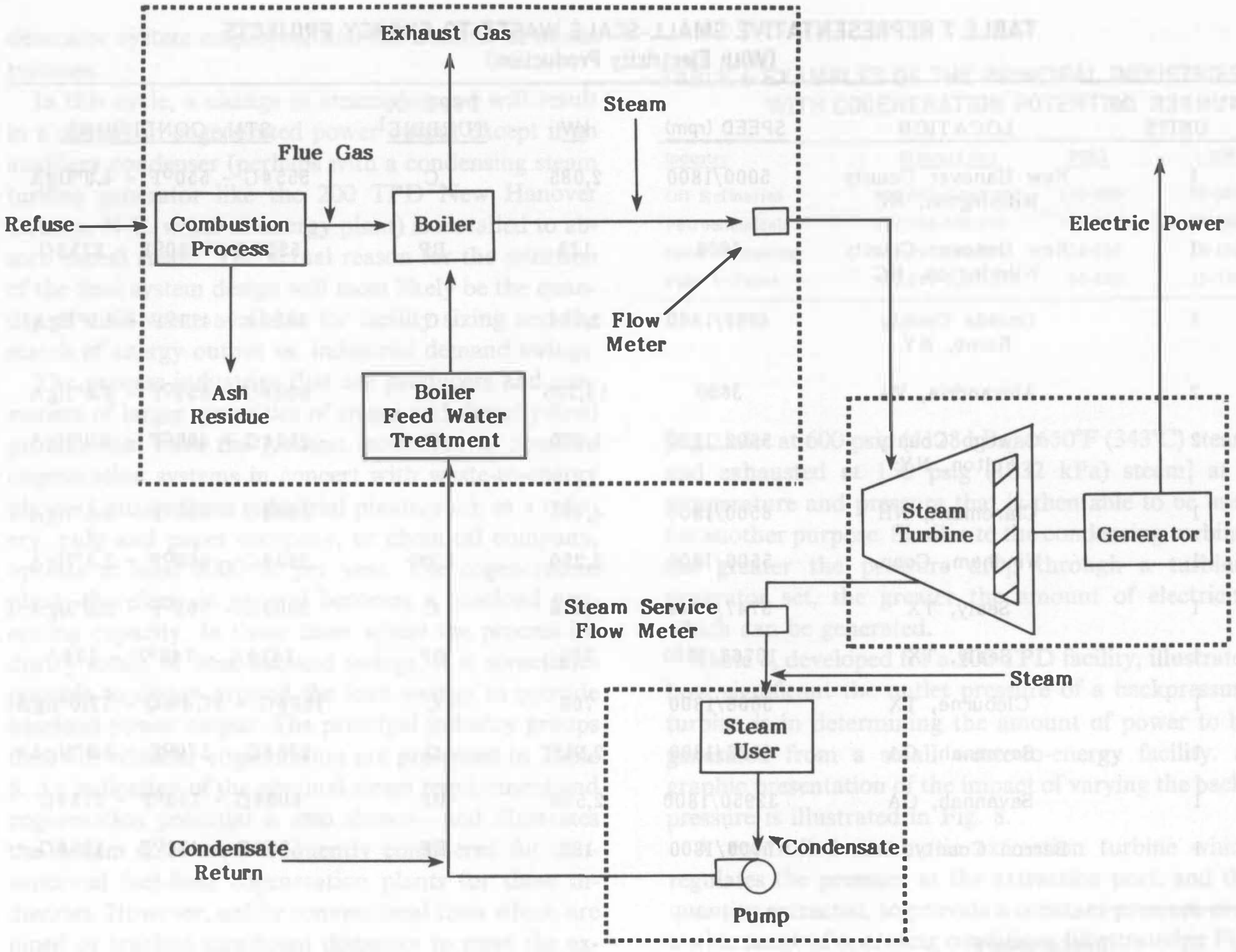

FIG. 7 BASIC COMPONENTS OF A WASTE-TO-ENERGY SYSTEM WITH A BACKPRESSURE TURBINE

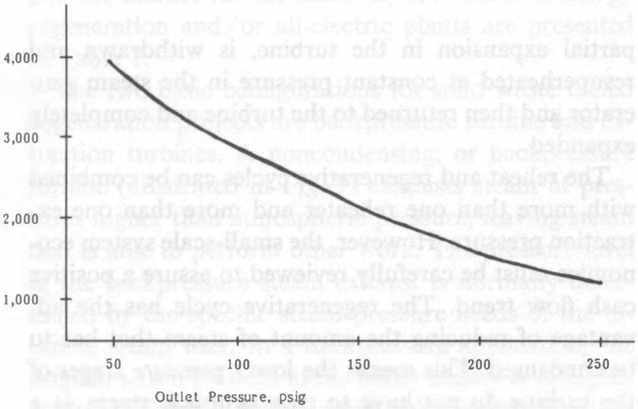

FIG. 8 POWER OUTPUT VS. VARIABLE TURBINE OUTLET PRESSURES

rates can be determined by use of a Willans line.[5] The Willans line is close to be being linear and usually is shown graphically as a straight line, so that its slope can be established by determining the full load steam flow rate and at least one other partial flow rate. In Fig. 10, the maximum throttle flow rate is assumed to be 60,000 lb (27,000 kg) per hour of steam, which equates to a nominal 300 TPD (270 tpd) system. The figure shows the plots of a Willans line commonly used with extraction type turbines. The lowest line represents a straight-through expansion turbine in which all

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[5] A Willans line is the steam flow rate (lb/hr) plotted against the power output (kW).

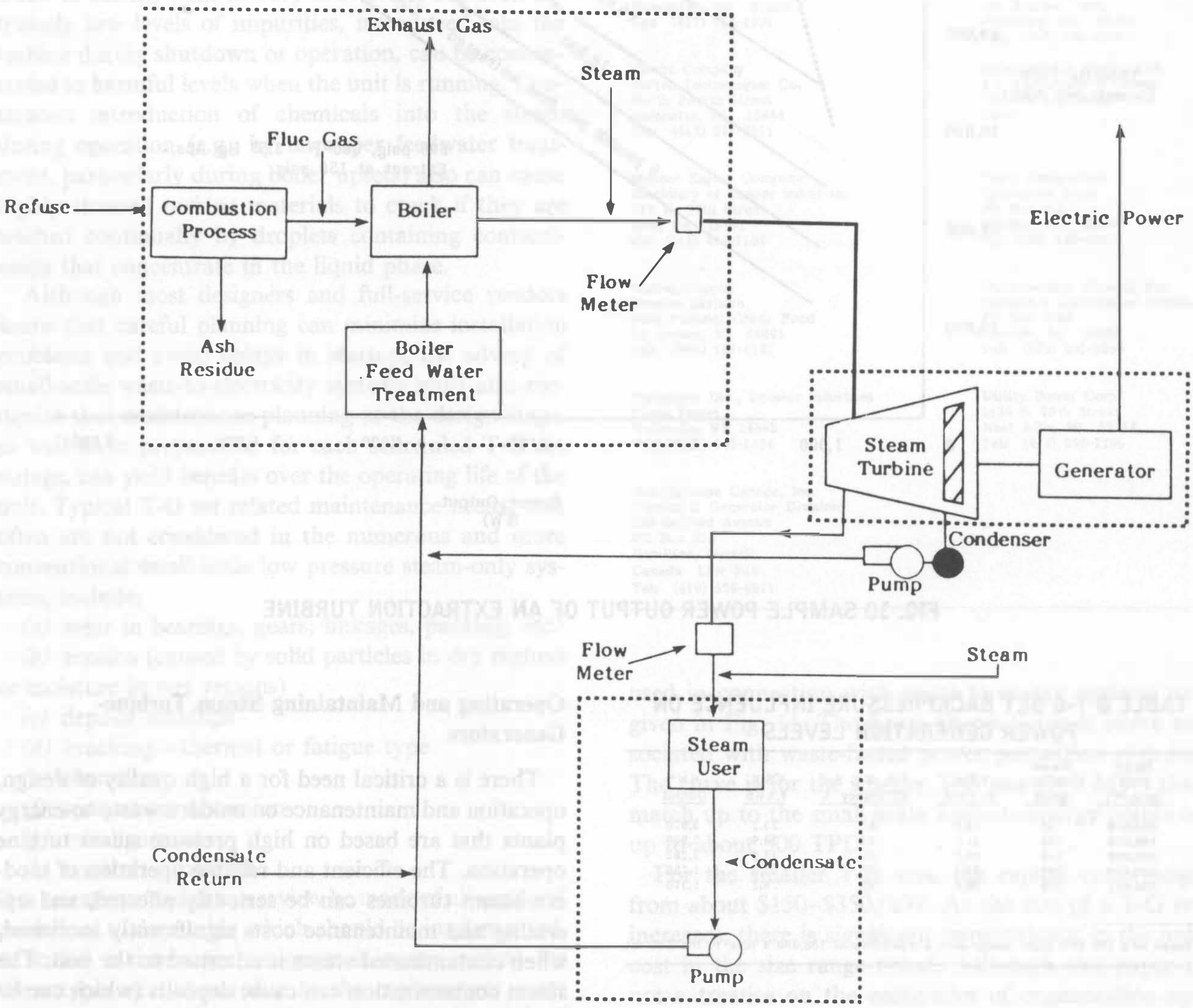

FIG. 9 BASIC COMPONENTS OF A WASTE-TO-ENERGY SYSTEM WITH AN EXTRACTION TURBINE

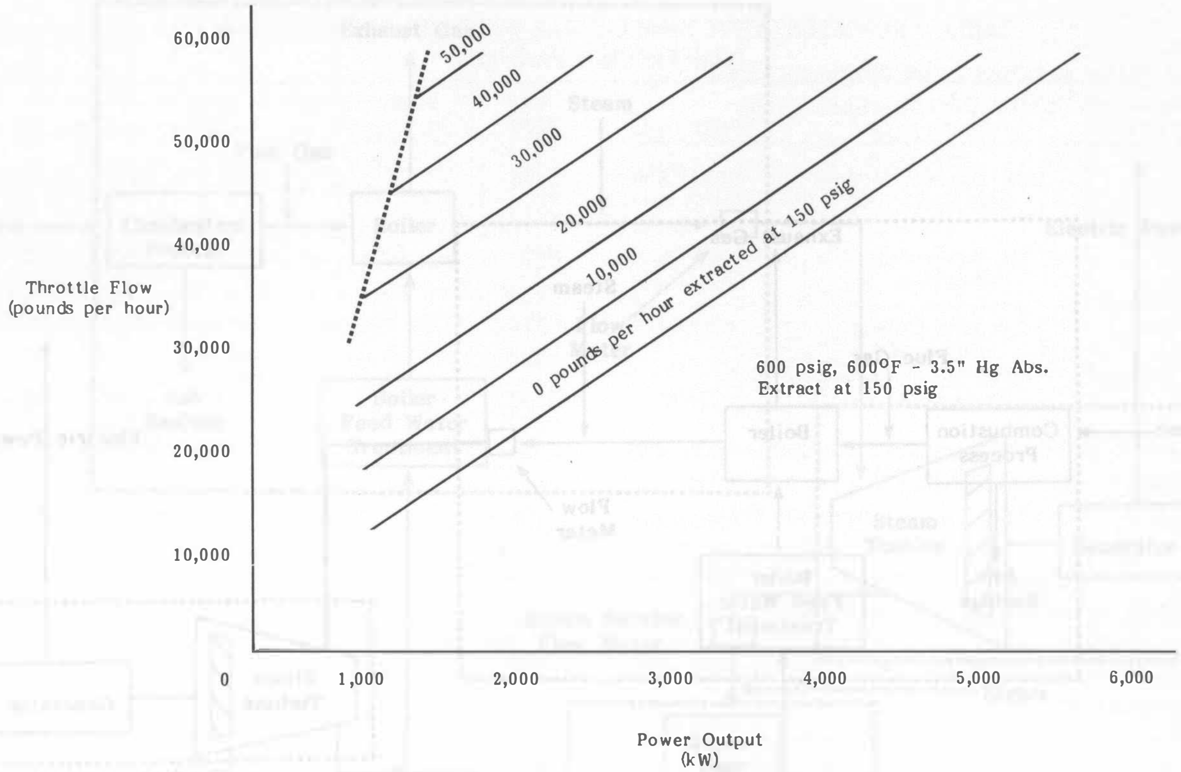

FIG. 10 SAMPLE POWER OUTPUT OF AN EXTRACTION TURBINE

TABLE 8 T-G SET BACKPRESSURE INFLUENCE ON POWER GENERATION LEVELS

Steam Inlet (psig,°F)	Steam Outlet (psig)	T.S.R. (#/kWh)	Est. T-G Set Efficiency %	A.S.R. #/kWh	KW* Output
600,650	50	16.9	67	25.2	3,970
600,650	100	21.7	64	33.9	2,950
600,650	150	26.7	61	43.8	2,285
600,650	200	32.5	57	57.0	1,750
600,650	250	39.6	52	76.2	1,310

* Based on a 500 TPD plant design with a steamflow of 100,000 #/hour to the T-G set.

of the steam enters at one pressure [600 psig (4128 kPa), 600°F (316°C)] and leaves at the selected exhaust point of 3.5 in. Hg Abs. (i.e. zero steam extraction at the 150 psig/1032 kPa designed extraction point). The other lines are representative of the changes in the amount of flow out through the 150 psig (1031 kPa) extraction point versus the total inlet throttle flow. The gross power output (kW) can be then read off the horizontal axis. As with all of the T-G set configurations, the actual steaming rate data and unit efficiencies can be supplied by the turbine manufacturers.

Operating and Maintaining Steam Turbine-Generators

There is a critical need for a high quality of design, operation and maintenance on modern waste-to-energy plants that are based on high pressure steam turbine operation. The efficient and reliable operation of modern steam turbines can be seriously affected, and operating and maintenance costs significantly increased, when contaminated steam is admitted to the unit. The steam contamination can cause deposits (which can be either inert or highly reactive, depending on the nature of the specific substances present) and can cause severe deterioration of the surfaces of steam path parts due to the abrasive action of the contaminants impacting on those parts. The specific problems resulting from the use of contaminated steam include:[6]

(a) stress—corrosion, cracking
(b) solid—particle erosion
(c) deposit building

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[6] Taken from "Power, Understanding the Observed Effects of Erosion and Corrosion in Steam Turbines," by Cowgill, T. and Robins, K. (Courtesy of General Electric Co.).

(d) peening or foreign material damage
(e) corrosive pitting

The need for good control of water chemistry, to prevent either deposition of solids or corrosion of steam-generator and turbine components, is generally recognized and the concentration of impurities in feedwater is usually held to very low levels, but even extremely low levels of impurities, introduced into the turbine during shutdown or operation, can be concentrated to harmful levels when the unit is running. Continuous introduction of chemicals into the steam during operation (e.g., by improper feedwater treatment, particularly during boiler upsets) also can cause highly stressed turbine materials to crack if they are washed continually by droplets containing contaminants that concentrate in the liquid phase.

Although most designers and full-service vendors know that careful planning can minimize installation problems and avoid delays in startup, the advent of small-scale waste-to-electricity systems must also recognize that maintenance planning at the design stage, as well as in preparation for each scheduled T-G set outage, can yield benefits over the operating life of the unit. Typical T-G set related maintenance needs, that often are not considered in the numerous and more conventional small-scale low pressure steam-only systems, include:

(a) wear in bearings, gears, linkages, packing, etc.
(b) erosion (caused by solid particles in dry regions or moisture in wet regions)
(c) deposit buildups
(d) cracking—thermal or fatigue type
(e) corrosion
(f) component distortion
(g) misalignment
(h) controls problems

It is recommended that a review be made of all vendors' guidelines to determine which should be implemented. A sound maintenance program is more critical for these types of systems due to the replacement costs and the potential for significant revenue shortfalls if operations are impaired.

Turbine Vendors

The names and addresses of representative turbine vendors that are actively marketing in the United States are given in Table 9.

TABLE 9 STEAM TURBINE VENDORS

BBC Brown Boveri & Co., Ltd.
PO Box 58
Ch-5401 Baden
Switzerland
U.S. Sales Office:
New Brunswick, NJ
Tel: (201) 731-6993

Carling Turbine Power
8 Nebraska St., PO Box 88
Worcester, MA 01613
Tel: (617) 752-2896

Coppus Engineering Corp.
344 Park Avenue
Worcester, MA 01610
Tel: (617) 756-8391

General Electric Company
Mechanical Drive Turbine Dept.
166 Boulder Drive
Fitchburg, MA 01420
Tel: (617) 343-1000

Elliott Company
United Technologies Co.
North Fourth Street
Jeannette, PA 15644
Tel: (412) 527-2811

Mitsubishi-Ce Heavy, Ltd.
5-1 Marunouchi 2 Chome
Chiyoda-Ku, Tokyo
Japan

Skinner Engine Company
Subsidiary of Banner Industries
337 W. 12th Street
Erie, PA 16501
Tel: (814) 454-7103

Terry Corporation
Lamberton Road
PO Box 555
Windsor, CT 06095
Tel: (203) 688-6211

Murray/Coppus
Process Division
3600 Pammel Creek Road
La Crosse, WI 54601
Tel: (608) 787-4181

Transamerica Delaval, Inc.
Turbine & Compressor Division
PO Box 8788
Trenton, NJ 08650
Tel: (609) 890-5000

Turbodyne Div., Dresser Industries
Coats Street
Wellsville, NY 14895
Tel: (716) 593-1234

Utility Power Corp.
1135 S. 70th Street
West Allis, WI 53214
Tel: (414) 256-1200

Westinghouse Canada, Inc.
Turbine & Generator Division
286 Sanford Avenue
PO Box 510
Hamilton, Ontario
Canada L8N 3K2
Tel: (416) 528-8811

COSTS

Representative information on the typical capital costs of turbine-generator (T-G) sets that would be used in connection with waste-to-energy systems are given in Fig. 11. The figure shows the cost curve associated with waste-fueled power generation systems. The curve is for the smaller T-G sets (1–9 MW) that match up to the small-scale waste-to-energy plants of up to about 500 TPD.

For the smaller T-G sets, the capital costs range from about \$150–\$350/kW. As the size of a T-G set increases, there is significant improvement in the unit cost in the size range noted. Although this paper is not a treatise on the economics of cogeneration opportunities from waste-to-energy plants, the long-term values and costs of the thermal energy components and electrical energy components must be evaluated fully.

SUMMARY

This paper is intended to be a short primer on the different types of power production systems (direct and cogeneration) for small-scale waste-to-energy plants. The application and review necessary for the possible implementation of a system would be no different than

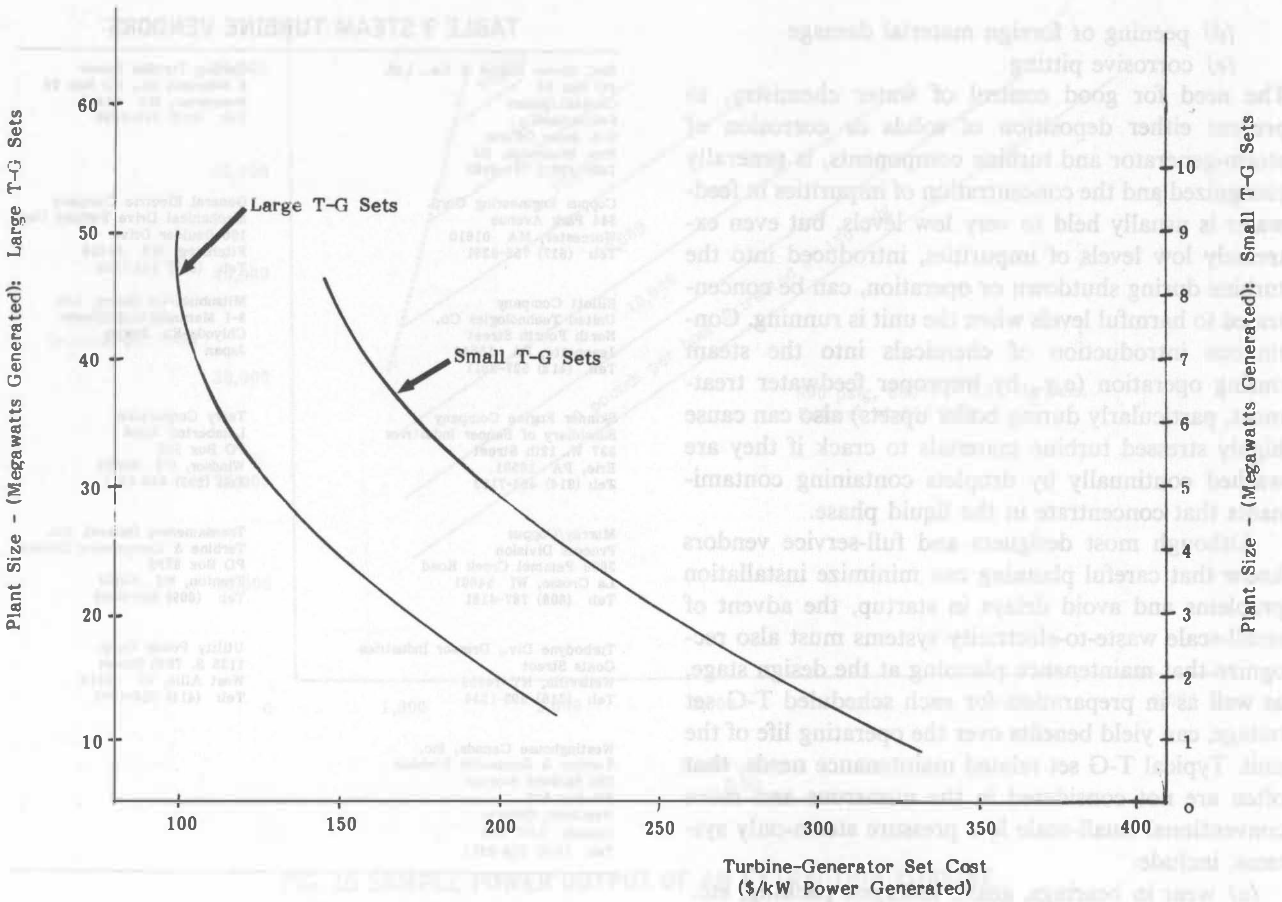

FIG. 11 UNIT COST CURVE FOR TURBINE-GENERATOR EQUIPMENT

any other solid waste-to-energy feasibility and procurement activity except that additional technical, marketing, and institutional issues pertaining to the power sales opportunity must be evaluated. Ancillary issues not discussed in this paper which also should be addressed include:

(a) obtaining the utility and cogeneration system data

(b) reviewing system size and performing energy analysis

(c) evaluating the economic feasibility

(d) performing the appropriate financial investment analysis

(e) selecting appropriate ownership and operating arrangements

(f) determining regulatory issues for the project

(g) conforming to air quality regulations

In this paper, a relatively simple overview to a complex subject has been presented. Therefore, it is not possible, nor our intent, to cover every specific option or T-G set configuration available in the marketplace today. However, the key point to remember is that unlike private industry which looks at 6 month to 2 year paybacks on equipment that is not product oriented, municipalities can afford to spend more capital funds on waste-to-energy projects because not only the first year costs, but the project's life-cycle costs for 15–20 years, are important considerations that go into making the final waste-to-energy implementation decision.

FORECASTS OF MSW TONNAGE—AN ADVANCED APPROACH

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ABSTRACT

This paper presents and demonstrates the use of multivariate regression analysis in forecasting MSW collection. It analyzes and quantifies the roles of population, employment and one-time events in the development of more accurate forecasting methods. Although the specific application is for San Francisco, California, a general discussion of the method allows the reader to apply it elsewhere.

PROBLEM STATEMENT

Refuse tonnage projections made for the City of San Francisco in 1981 were found to be significantly different from actual tonnage data in 1983 and 1984.

Table 1 shows: *(a)* the recorded tonnages from 1972 through 1981, which were used as the basis for the aforementioned projections; *(b)* those projections, for 1982 through 1984; and *(c)* actual tonnages, 1982 through 1984. In the 3 year period from 1981 to 1984, the actual tonnage grew to exceed the projection by more than 11%.

BACKGROUND

In 1979, the San Francisco Board of Supervisors established the Solid Waste Management Program as a group of planning, engineering, legal and financial specialists reporting to the City's Chief Administrative Officer. The planning functions of the Program soon required projections of refuse tonnage.

In 1981, an effort was made to develop such projections through a detailed review and statistical analysis of available tonnage records. The output from this task, the equation below, provided the best fit of an exponential-growth curve to the available data. It estimated an unusually low growth rate of 0.2%/year. After considering the City's limited room for growth and stable population, it was decided to use this growth rate in projecting future waste quantities.

$$\text{tonnage} = 467 \times (1.0021)^{(\text{year}-1900)} \times \exp(-0.101f)$$

where

$f = 1.0$ for years 1972 through 1977
$f = 0.5$ for year 1978
$f = 0$ for years 1979 through 1981

STRATEGY

Staff of the Solid Waste Management Program were faced with a significant discrepancy between predicted and actual data, occurring within 3 years of the prediction. They were also faced with the need for a re-

TABLE 1 REFUSE TONNAGE DATA AND 1981 PROJECTIONS
Source: City of San Francisco, Solid Waste Management Program

	MEASURED TONS, PRE 1981	PROJECTIONS	MEASURED TONS, POST 1981
1972	488,700		
1973	514,600		
1974	493,400		
1975	499,700		
1976	484,800		
1977	496,200		
1978	529,200		
1979	546,300		
1980	551,400		
1981	556,500		
1982		557,613	574,600
1983		558,728	609,300
1984		559,846	625,600

liable 20 year forecast of refuse quantity to support their planning and development efforts. After discussion among staff, City consultants, and others, it was decided that the most fruitful approach would be to use an *explanatory* rather than *extrapolative* model. That is, rather than apply the earlier strategy of simply extending the curve that best fit the tonnage data, a four-step technique would be applied:

(a) Identify well-documented independent variables (such as population) which could be expected to have a causal relationship to refuse quantity.

(b) Using multivariate least-squares regression analysis, derive a linear equation (the "regression equation") which can be used to estimate refuse tonnage based on the values of the independent variables. A description of this method is given in the Appendix to this paper.

(c) Find or develop predictions of the independent variables for future years.

(d) Using the regression equation found in step 2 and the predicted values of the independent variables, calculate projected tonnages for future years.

PROCEDURE

Regression Analysis on Selected Independent Variables

A broad list was developed of quantifiable variables (Table 2) that were expected to have a causal relationship to refuse tonnage. Research was then done to find data for each variable, specific to San Francisco, for the period 1971 to 1984. As this research was done, it became clear that for many of the variables, data were not available for every year, and that for some, useful 20 year projections did not exist.

With this in mind, the first few regression analyses were done using variables that offered reliable year-by-year historical data and that have well-established forecasting methods. Each trial regression resulted in a linear equation having tonnage as a dependent variable and several terms, each consisting of an independent or explanatory variable and an estimated coefficient, as components. Table 3 shows an example.

The following discussion refers to Table 3. The value "REFUSE" is the predicted tonnage of San Francisco refuse for a given year. On the right side of the equation, the variables "POP" and "EMP" represent the population and employment data available for that year. Each is multiplied by regression coefficients calculated using the methods described in the Appendix: 2.413 and 1.353, respectively. In words, the equation states that each additional member of the population or employed labor force will generate 2.413 tons (2.189 t) and 1.353 tons (1.227 t) of refuse per year, respectively. Finally, there is an "intercept" term which states that if all other variables were equal to zero, 1847 tons (1675 t) per year would be generated. The intercept has little practical value in a forecasting application.

It should be noted that the signs on the two regression coefficients are both positive, indicating (as ex-

TABLE 2 VARIABLES CHOSEN FOR INVESTIGATION

POPULATION
RESIDENTIAL HOUSING STOCK
OFFICE SPACE IN DOWNTOWN AREA
NUMBER OF WORKERS
COMMUTER TRAFFIC INTO SAN FRANCISCO
HOTEL ROOMS AND OCCUPANCY
EFFECT OF SEVERE WINTERS (81-82, 82-83)
CLOSURE OF PORT-AREA DEBRIS FILL (1978)
TOTAL NUMBER OF VISITORS
WHOLESALE AND RETAIL ESTABLISHMENTS: NUMBER AND REVENUE
NUMBERS OF EMPLOYEES, BY INDUSTRY TYPE
BUSINESS TAXES: PAYROLL AND GROSS RECEIPTS

pected) that more people would produce more refuse. Had they been negative, it would have been necessary to rethink the approach because the equation would be nonsensical and likely to be misleading about the future.

In Table 3, the numbers in parentheses are values of the "Students-*t*" distribution. This statistic provides a test of whether a coefficient is significantly different from zero. As a rule of thumb, a *t*-statistic whose absolute value exceeds two indicates a coefficient that is considered to be highly significant.

"Adjusted *R*-squared" indicates the amount of variation in REFUSE that is accounted for by EMP and POP. In general, the "adjusted *R*-squared" value may vary between zero and one, where zero would indicate that none of the variation in REFUSE has been accounted for by the explanatory variables, and 1.0 would indicate that all of the variation has been accounted for. The actual value of 0.917 indicates that 91.7% of the variation in REFUSE has been accounted for by POP and EMP.

The Durbin-Watson statistic indicates whether the difference between actual and fitted values of the equation vary randomly over time. For the number of prior years and the number of independent variables being considered here, an estimate greater than 2.0 is considered a good indicator of random variation. If the pattern is not random, the *t*-statistics will overestimate the true significance of the regression coefficients.

Several regression equations were estimated using various combinations of explanatory variables. The variables and their associated statistics are shown in Table 4. As is apparent, the best fit was found using the population and employment variables. Due to limits on project time and resources, and the good fit found with these explanatory variables, no further combination from the list in Table 2 was explored.

An improved fit to the data was achieved in the following way. It was known that disposal tonnage had increased dramatically in 1977 and 1978, coinciding with the closure of an inert-waste landfill (the Islais Creek landfill) near the Port of San Francisco. To account for this change, a dummy variable having the value 0 prior to 1978 and 1 thereafter was introduced, and an analysis was run to represent this once-and-for-all change in addition to the influences of population and employment. The results, shown in Table 5, were a significant improvement in fit. For the purpose of this study, then, the equation in Table 5 was considered the best solution. Table 6 and Fig. 1 compare actual historical data with tonnage data calculated using this equation.

Projections of Explanatory Variables

The projections of population and employment used in this study are both by the Association of Bay Area Governments (ABAG), a well-established regional planning organization. In the case of employment, an adjustment was necessary, inasmuch as the historical employment data were from December of each year, whereas the forecasts are annual averages. On the basis of a sample of several recent years, it appeared that December employment was consistently about 2% higher than annual average employment. Thus, in making the REFUSE forecasts, the forecasted employment was multiplied by 1.02 so that the forecasts would be consistent with the historical data.

Forecasts of Refuse Tonnage

Using the equation from Table 5, and the projections from ABAG, a year-by-year forecast of refuse tonnage was produced. It is shown in Table 7 and Fig. 2.

TABLE 3 SAMPLE CORRELATION EQUATION

REFUSE =	2.413 POP +	1.353 EMP -	1847
	(5.76)	(11.62)	(-5.67)

Adjusted R-squared = 0.917

Durbin-Watson statistic = 2.158

TABLE 4 RESULTS OF CORRELATION ANALYSES

	Equations	R squared	Durbin-Watson
1.	REFUSE = 1.015 * EMPLOYMENT +3.87	0.675	0.587
2.	REFUSE = 0.857 * EMPLOYMENT -0.0952 * REVENUE + 521.97	0.756	1.524
3.	REFUSE = 0.6489 * EMPLOYMENT +0.0852 * TAX -26.06	0.784	1.822
4.	REFUSE = 2.413 * POPULATION +1.353 * EMPLOYMENT -1846.9	0.917	2.158

TABLE 5 FINAL CORRELATION EQUATION

REFUSE =	2.425 POP +	0.898 EMP +	36.623 D -	1635.758
	(6.95)	(4.11)	(2.33)	(-5.72)

Adjusted R-squared = 0.94

Durbin-Watson statistic 2.41

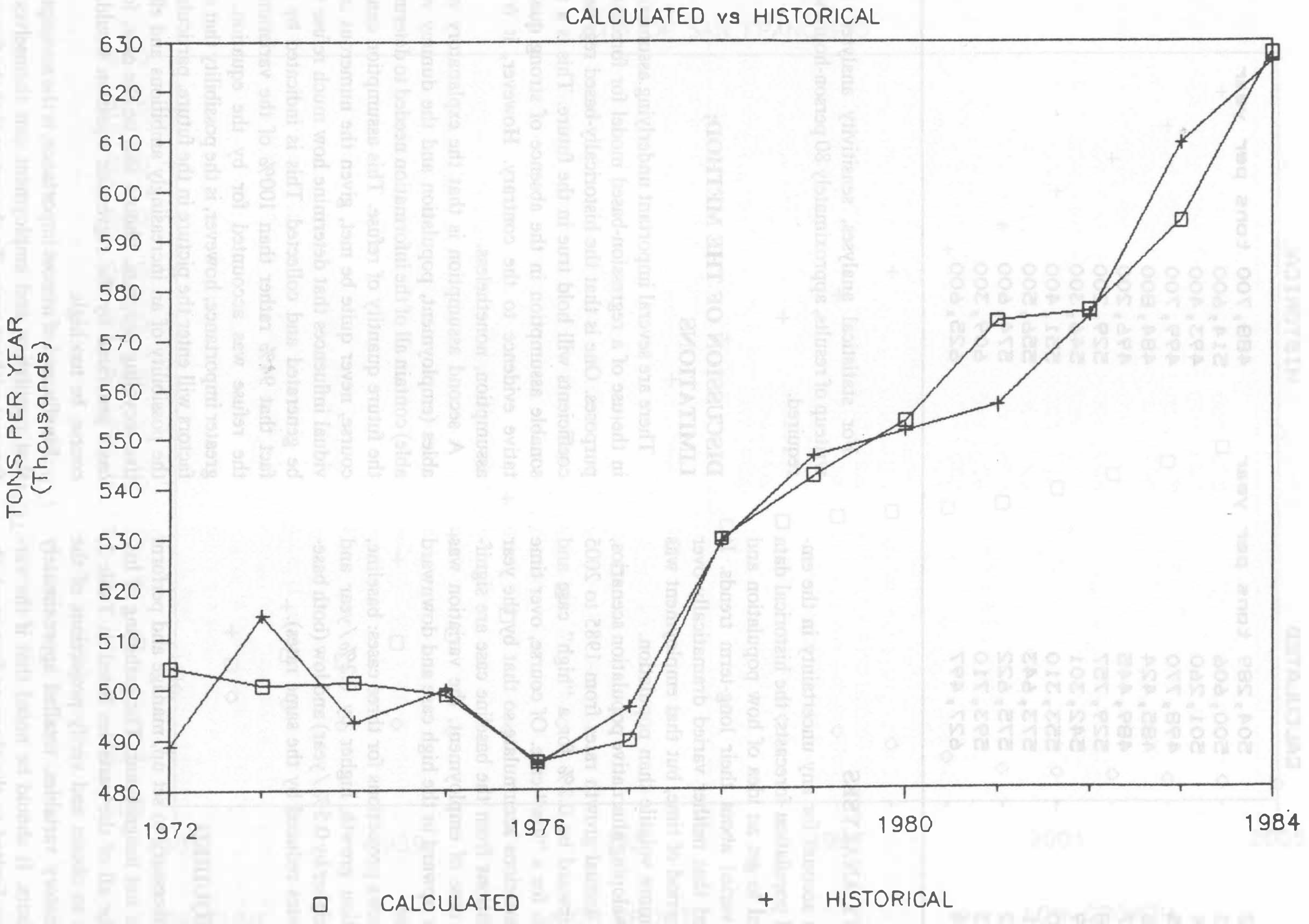

FIG. 1 COMPARISON OF TONNAGE DATA

TABLE 6 COMPARISON OF CALCULATED AND HISTORICAL TONNAGE DATA

	CALCULATED	HISTORICAL
1972	504,289 tons per year	488,700 tons per year
1973	500,606	514,600
1974	501,260	493,400
1975	498,770	499,700
1976	485,424	484,800
1977	489,445	496,200
1978	529,757	529,200
1979	542,301	546,300
1980	553,310	551,400
1981	573,643	556,500
1982	575,622	574,600
1983	593,710	609,300
1984	627,497	625,600

SENSITIVITY ANALYSES

In order to account for any uncertainty in the employment and population forecasts, the historical data were examined to get an idea of how population and employment varied about their long-term trends. It was discovered that neither varied dramatically over an extended period of time, but that employment was considerably more volatile than population.

Thus, in developing alternative population scenarios, the projected annual growth rates from 1985 to 2005 were varied upward by 0.2% for a "high" case and down by 0.2% for a "low" case. Of course, over time these small variations accumulate so that by the year 2005 the deviations from the baseline case are significant. In the case of employment, the variation was 0.5% per year upward in the high case and downward in the low case.

Figure 2 shows projections for three cases: baseline, high (population growth higher by 0.2%/year and employment higher by 0.5%/year) and low (both baseline growth rates reduced by the same rates).

EFFORT REQUIRED

The work necessary to set up, manage and perform this study was not insignificant. The gathering of historical data for all of the variables listed in Table 2, plus the time to obtain and verify projections of the chosen explanatory variables, totalled approximately 120 person-hours. It should be noted that if the variables had been limited to the three or four most obvious ones at the outset, much less research time would have been needed.

For statistical analyses, sensitivity analyses, and writeup of results, approximately 80 person-hours were required.

DISCUSSION OF THE METHOD; LIMITATIONS

There are several important underlying assumptions in the use of a regression-based model for forecasting purposes. One is that the historically-based regression coefficients will hold true in the future. This is a reasonable assumption in the absence of strong quantitative evidence to the contrary. However, it *is* an assumption, nonetheless.

A second assumption is that the explanatory variables (employment, population and the dummy variable) contain all of the information needed to determine the future quantity of refuse. This assumption can, of course, never quite be met, given the numerous individual influences that determine how much refuse will be generated and collected. This is indicated by the fact that 94% rather than 100% of the variation in the refuse was accounted for by the equation. Of greater importance, however, is the possibility that new factors will enter the picture in the future, particularly the possibility of an increasingly ambitious and effective recycling program. Should this be the case, forecasts generated by the regression equation would, of course, be too high.

Finally, and of utmost importance, is the assumption that population and employment can themselves be forecast without error. To the extent that the forecasts of these variables are wrong, forecasts of refuse based on this model will be wrong also.

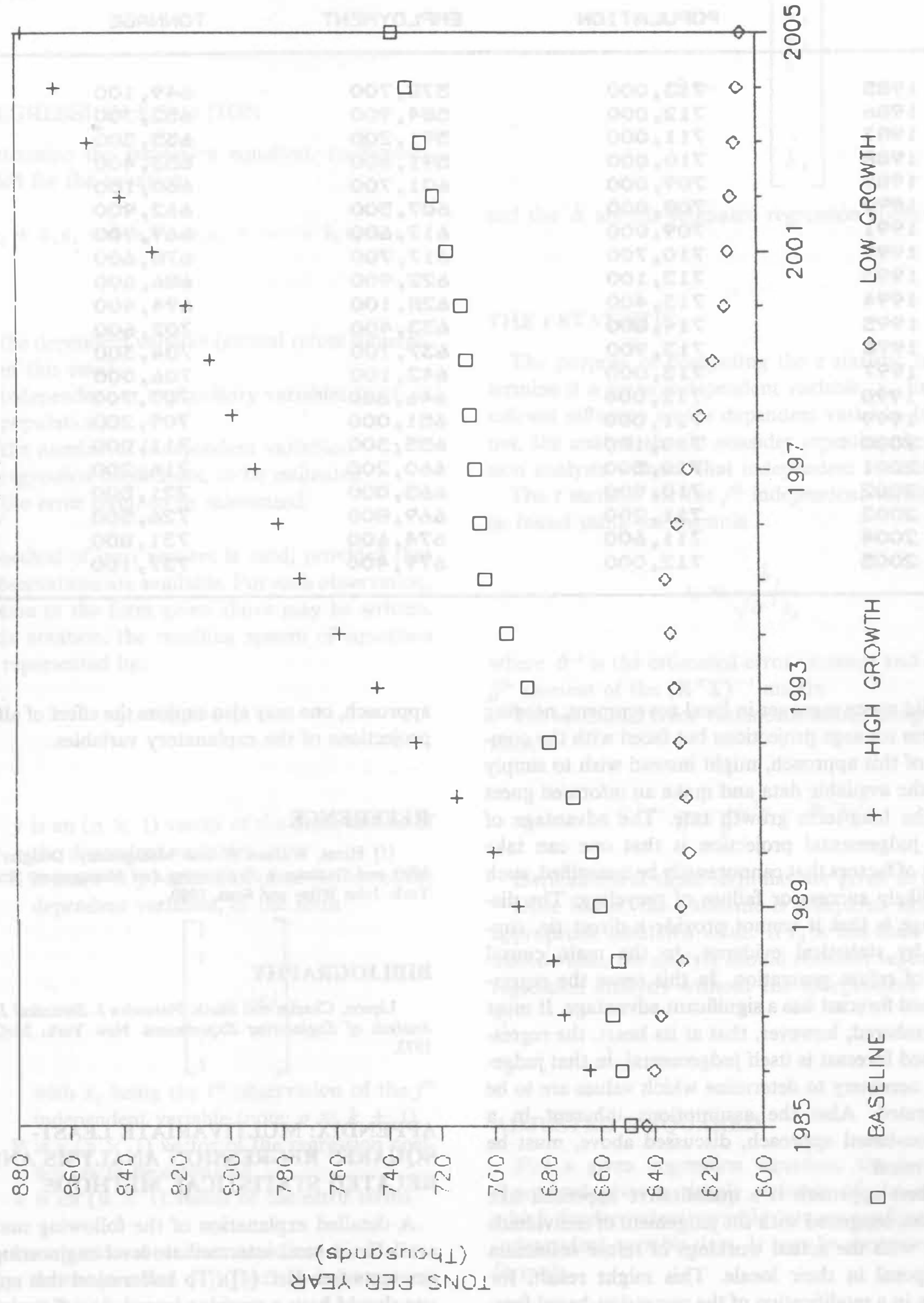

FIG. 2 PROJECTIONS OF REFUSE TONNAGE

TABLE 7 PROJECTIONS OF REFUSE TONNAGE

	POPULATION	EMPLOYMENT	TONNAGE
1985	713,000	578,700	649,100
1986	712,000	584,900	652,300
1987	711,000	591,200	655,500
1988	710,000	591,600	653,400
1989	709,000	601,700	660,100
1990	708,000	607,500	662,900
1991	709,000	612,600	669,900
1992	710,700	617,700	678,600
1993	712,100	622,900	686,600
1994	713,400	628,100	694,400
1995	714,800	633,400	702,600
1996	713,900	637,700	704,300
1997	713,000	642,100	706,000
1998	712,000	646,600	707,700
1999	711,000	651,000	709,200
2000	710,100	655,500	711,000
2001	710,500	660,200	716,200
2002	710,900	665,000	721,500
2003	711,200	669,800	726,500
2004	711,600	674,600	731,800
2005	712,000	679,400	737,100

A solid waste manager in local government, needing long-term tonnage projections but faced with the complexity of this approach, might instead wish to simply review the available data and make an informed guess about the long-term growth rate. The advantage of such a judgemental projection is that one can take account of factors that cannot easily be quantified, such as the likely success or failure of recycling. The disadvantage is that it cannot provide a direct tie, supported by statistical evidence, to the main causal factors of refuse generation. In this sense the regression-based forecast has a significant advantage. It must be remembered, however, that at its heart, the regression-based forecast is itself judgemental, in that judgement is necessary to determine which values are to be incorporated. Also the assumptions inherent in a regression-based approach, discussed above, must be kept in mind.

The best approach is a quantitative approach like regression, tempered with the judgement of individuals familiar with the actual workings of refuse collection and disposal in their locale. This might result, for example, in a modification of the regression-based forecast based on the knowledge or informed opinion of how effective recycling is expected to be. With this approach, one may also explore the effect of alternative projections of the explanatory variables.

REFERENCE

[1] Hines, William W. and Montgomery, Douglas C. *Probability and Statistics in Engineering And Management Science.* New York: John Wiley and Sons, 1980.

BIBLIOGRAPHY

Lipson, Charles and Sheth, Narendra J. *Statistical Design and Analysis of Engineering Experiments.* New York: McGraw-Hill, 1973.

APPENDIX: MULTIVARIATE LEAST-SQUARES REGRESSION ANALYSIS AND RELATED STATISTICAL METHODS

A detailed explanation of the following material is available in most intermediate-level engineering statistics texts (see Ref. [1]). To understand this approach, one should have a working knowledge of statistics and matrix mathematics. Many computer software packages are available to perform such analysis. In this particular case, the authors used Micro-TSP on an IBM PC-XT.

THE REGRESSION EQUATION

To determine the regression equation, coefficients are needed for the equation:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_kx_k + e,$$

where

y = the dependent variable (annual refuse tonnage, in this case)

x_i = independent or explanatory variables (such as population)

k = the number of independent variables

b_i = regression coefficients, to be estimated

e = the error term, to be minimized

The method of least squares is used, provided that $n > k$ observations are available. For each observation, an equation in the form given above may be written. In matrix notation, the resulting system of equations may be represented by:

$$\mathbf{y} = \mathbf{X}\,\mathbf{B} + \mathbf{e}$$

where

y is an $(n \times 1)$ vector of the observations of the dependent variables

X is an $(n \times p)$ matrix of observations of the dependent variables, of the form

$$\begin{bmatrix} 1 & \\ 1 & \\ \cdot & |x_{ij}| \\ \cdot & \\ \cdot & \\ 1 & \end{bmatrix}$$

with x_{ij} being the i^{th} observation of the j^{th} independent variable (note: $p = k + 1$)

B is a $(p \times 1)$ vector of the regression coefficients to be estimated

e is an $(n \times 1)$ vector of the error terms

This vector equation must then be solved for **B**. It can be shown (see Ref. [1]) that the least squares estimator of **B** is

$$\hat{\mathbf{B}} = (\mathbf{X}^T\mathbf{X})^{-1}\,\mathbf{X}^T\mathbf{y}$$

where

$$\hat{\mathbf{B}} = \begin{bmatrix} \hat{b}_0 \\ \hat{b}_1 \\ \hat{b}_2 \\ \cdot \\ \cdot \\ \cdot \\ \hat{b}_k \end{bmatrix}$$

and the $\hat{b}_i$ are the estimated regression coefficients.

THE t STATISTIC

The purpose of computing the t statistic is to determine if a given independent variable, x_j, has a significant influence on the dependent variable. If it does not, the analyst should consider repeating the regression analysis without that independent variable.

The t statistic for the j^{th} independent variable may be found using the formula

$$t_0 = \frac{\hat{b}_j}{\sqrt{\hat{\sigma}^2\, c_{jj}}}$$

where $\hat{\sigma}^2$ is the estimated error variance and c_{jj} is the jj^{th} element of the $(\mathbf{X}^T\mathbf{X})^{-1}$ matrix.

The estimated error variance is found using the formula

$$\hat{\sigma}^2 = \frac{1}{n-p}\,\mathbf{y}^T\mathbf{y} - \hat{\mathbf{B}}^T\mathbf{X}^T\mathbf{y}$$

Derivations of these formulas are given in Ref. [1]. Once found, the t statistic is compared against the appropriate tabulated value. If t_0 is less than the tabulated value, the analyst should consider repeating the regression analysis without the independent variable x_j.

ADJUSTED R-SQUARED

For a given regression equation, the *un*adjusted *R*-squared gives a simple estimate of the extent to which the dependent-variable data are predicted by the independent-variable data. It may be computed by the formula

$$R^2 = 1 - \frac{(\hat{\sigma}^2)(n-p)}{S_{yy}}$$

where $\hat{\sigma}^2$ is the estimated error variance (see formula in preceding discussion of t-statistic), and

$$S_{yy} = \mathbf{y}^T\mathbf{y} - \frac{(\Sigma\, y_i)^2}{n}$$

The unadjusted R-squared provides a useful method of comparing regression equations to choose the most accurate set of predictive variables. However, it may fail to indicate the best set when comparing equations with differing numbers of independent variables. The adjusted R-squared takes the number of variables into account; it is found using the formula

$$R^2 = 1 - \frac{n-1}{n-p}(1-R)$$

THE DURBIN-WATSON STATISTIC

Many regression-analysis problems use data points that have been taken over intervals of time; that is, the sets $(y_i, x_{i1}, x_{i2}, \cdots x_{ik})$ may represent values for the i^{th} year. A regression equation developed from such data may have error terms which are not strictly random; they may, for example, increase or decrease with time. In this type of situation, the error terms are said to be *autocorrelated* variables. When this occurs, the regression equation found by the methods outlined above does not produce the best estimates, i.e., those with minimum variance in error.

The Durbin-Watson statistic provides a way to test for autocorrelation. It is found using the formula

$$D = \frac{\sum_{i=2}^{n}(e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i^2}$$

This statistic may be compared with tabulated values (see Ref. [1]) to determine if autocorrelation is: *(a)* definitely occurring; *(b)* definitely not occurring; or *(c)* possibly occurring (more data needed to confirm). In general, where four or fewer independent variables are involved and the number of observations is 12 or more, a Durbin-Watson statistic greater than 2.0 provides a high level of confidence that the error terms are not autocorrelated.

APPLICATION

For the problem which is the subject of this paper, the methods described above were applied using the following input data:

X Matrix

From available records of population and employment, and using a "dummy" variable to represent closure of the Islais Creek landfill:

$$\mathbf{X} = \begin{array}{c} \begin{array}{cccc} & \text{POP} & \text{EMP} & \text{DUMMY} \end{array} \\ \begin{bmatrix} 1 & 709300 & 467700 & 0 \\ 1 & 705300 & 474400 & 0 \\ 1 & 702200 & 483500 & 0 \\ 1 & 697100 & 494500 & 0 \\ 1 & 691300 & 495300 & 0 \\ 1 & 686700 & 512200 & 0 \\ 1 & 683000 & 526300 & 1 \\ 1 & 680100 & 548100 & 1 \\ 1 & 678900 & 563600 & 1 \\ 1 & 684100 & 572200 & 1 \\ 1 & 691100 & 555500 & 1 \\ 1 & 698300 & 556200 & 1 \\ 1 & 707900 & 567900 & 1 \end{bmatrix} \end{array}$$

From the available historical data on refuse tonnage (see Table 1):

$$\mathbf{y} = \begin{bmatrix} 488700 \\ 514600 \\ 493400 \\ 499700 \\ 484400 \\ 496200 \\ 529200 \\ 546300 \\ 551400 \\ 556500 \\ 574600 \\ 609300 \\ 625600 \end{bmatrix}$$

These data were used to calculate the $\hat{\mathbf{B}}$ matrix, which provides the coefficients of the equation in Table 5, and related statistical parameters.